Econometrics (ECON4160)
PcGive lectures

by Øyvind Eitrheim

Research department, C51,
Norges Bank (The Central Bank of Norway),
Box 1179 Sentrum, N-0107 Oslo,
Phone: +4722316161,
Fax: +4722424062,
Email: oyvind.eitrheim@norges-bank.no

PcGive lectures

Where and when

- 8 PC-based lectures
  Time: 8.15-10.00 (September-November 2009)
  Fridays: 4 Sep, 11 Sep, 18 Sep, 25 Sep, -, 9 Oct, 16 Oct, 23 Oct, 6 Nov (PC-room)

Main topics

- Introduction to OxMetrics software (GiveWin, PcGive, PcFiml, PcNaive, Ox, Oxrun, OxEdit)
  Practical examples using PcGive for simple econometric analysis
  Selected topics in econometrics

1. lecture

- Introduction
  - Where and when
  - Main topics
  - Model types
    * Bivariate regression models
    * Multiple regression models
    * Systems of linear equations
- Introduction to OxMetrics software (GiveWin, PcGive, PcFiml, PcNaive, Ox, Oxrun, OxEdit)
- Example using data for inflation in Norway
  - Disposition
  - The files for this course
  - Interactive use of GiveWin and PcGive
  - Introduction to batch files
  - Introduction to the batch language in GiveWin and PcGive

Main types of models

- Linear regression models with one single explanatory variable,

$$y_t = \alpha + \beta x_t + \varepsilon_t$$

Stochastic regressors x_t (consider the joint distribution of (y_t, x_t))
Orthogonality condition ($Cov(x_t, \varepsilon_t) = 0$)
Normality condition ($\varepsilon_t \simeq N(0, \sigma^2)$)
Properties of OLS- and IV estimators
Diagnostic tests, R^2, DW, $\hat{\sigma}$ etc.

Interpreting the linear regression model as conditional expectation

Let the DGP for $(y_t, x_t)'$ be given by the simultaneous Normal probability density function

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} \sim \text{Niid}\left(\begin{bmatrix} \mu_y \\ \mu_x \end{bmatrix}, \underbrace{\begin{bmatrix} \sigma_{yy} & \sigma_{yx} \\ \sigma_{xy} & \sigma_{xx} \end{bmatrix}}_{\Sigma} \right)$$

where μ_y and μ_x are the (unconditional) expected values of y_t and x_t, and Σ is the joint covariance matrix for $(y_t, x_t)'$.

Consider the conditional distribution of

$$(y_t|x_t) \sim \text{Niid}\left(\underbrace{\beta_0 + \beta_1 x_t}_{\mu_{y|x}}, \sigma^2_{y|x} \right)$$

where the conditional expectation and variance of $(y_t|x_t)$ are given by

$$\mu_{y|x} = \underbrace{\mu_y - \frac{\sigma_{xy}}{\sigma_{xx}}\mu_x}_{\beta_0} + \underbrace{\frac{\sigma_{xy}}{\sigma_{xx}}}_{\beta_1} x_t$$

and

$$\sigma^2_{y|x} = \sigma_{yy} - \sigma_{yx}\sigma^{-1}_{xx}\sigma_{xy} \leq \sigma_{yy}$$

respectively, and the regression model can be written as

$$\begin{aligned} y_t &= \beta_0 + \beta_1 x_t + \varepsilon_t \\ (\varepsilon_t|x_t) &\sim \text{Niid}\left(0, \sigma^2_{y|x}\right) \\ \beta_0 &= \mu_y - \beta_1\mu_x \\ \beta_1 &= \frac{\sigma_{xy}}{\sigma_{xx}} \end{aligned}$$

Using matrix notation we can write

$$\underset{T\times 1}{y} = \underset{T\times 2}{[\iota \vdots x]} \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} = \underset{T\times 2}{X} \underset{2\times 1}{\beta} + \underset{T\times 1}{\varepsilon}, \qquad \varepsilon \sim \mathsf{N}(0, \sigma^2 I_T)$$

$E[X'\varepsilon] = 0;$ assuming independence between errors and RHS-variables (orthogonality)

- Multiple regression models,

$$y_t = \alpha + \beta_1 x_{1t} + \beta_2 x_{2t} + \varepsilon_t$$

Multicollinearity
Omitted variables
Residual regression
Tests of linear restrictions

Multiple regression analysis - an example

Let the DGP for (y_t, x_{1t}, x_{2t}) be given by

$$\begin{pmatrix} y_t \\ x_{1t} \\ x_{2t} \end{pmatrix} \simeq \text{Niid}\left(\begin{bmatrix} \mu_y \\ \mu_1 \\ \mu_2 \end{bmatrix}, \underbrace{\begin{bmatrix} \sigma_{yy} & \sigma_{y1} & \sigma_{y2} \\ \sigma_{1y} & \sigma_{11} & \sigma_{12} \\ \sigma_{2y} & \sigma_{21} & \sigma_{22} \end{bmatrix}}_{\Sigma} \right)$$

We consider the conditional distribution of

$$(y_t \mid x_{1t}, x_{2t}) \simeq \text{Niid}(\beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t}, \sigma^2)$$

where

$$\sigma^2 = \sigma_{yy} - (\sigma_{y1} \sigma_{y2}) \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \begin{pmatrix} \sigma_{1y} \\ \sigma_{2y} \end{pmatrix} \leq \sigma_{yy}$$

We have

$$\beta_0 = \mu_y - \beta_1\mu_1 - \beta_2\mu_2$$

$$\beta = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}^{-1} \begin{pmatrix} \sigma_{1y} \\ \sigma_{2y} \end{pmatrix}$$

$$= \frac{1}{\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21}} \begin{pmatrix} \sigma_{22} & -\sigma_{21} \\ -\sigma_{12} & \sigma_{11} \end{pmatrix} \begin{pmatrix} \sigma_{1y} \\ \sigma_{2y} \end{pmatrix}$$

$$\beta_1 = \frac{\sigma_{1y} - \frac{\sigma_{21}}{\sigma_{22}}\sigma_{2y}}{\sigma_{11} - \frac{\sigma_{21}}{\sigma_{22}}\sigma_{12}}$$

$$\beta_2 = \frac{\sigma_{2y} - \frac{\sigma_{12}}{\sigma_{11}}\sigma_{1y}}{\sigma_{22} - \frac{\sigma_{12}}{\sigma_{11}}\sigma_{21}}$$

The conditional regression model (DGP)

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \varepsilon_t, \quad (\varepsilon_t \mid x_{1t}, x_{2t}) \simeq \text{Niid}(0, \sigma^2)$$

We can reparameterize the model

$$
\begin{aligned}
y_t &= \beta_0 + \beta_1 x_{1t} - \beta_1 x_{2t} + \beta_1 x_{2t} + \beta_2 x_{2t} + \varepsilon_t \\
&= \beta_0 + \beta_1 \underbrace{(x_{1t} - x_{2t})}_{x_{12t}} + (\beta_1 + \beta_2) x_{2t} + \varepsilon_t \\
&= \beta_0 + \beta_1 x_{1t} - \beta_2 x_{1t} + \beta_2 x_{1t} + \beta_2 x_{2t} + \varepsilon_t \\
&= \beta_0 + (\beta_1 + \beta_2) x_{1t} + \beta_2 \underbrace{(x_{2t} - x_{1t})}_{x_{21t}} + \varepsilon_t \\
&= \beta_0 + \beta_1 x_{1t} - \beta_1 x_{2t} + \beta_1 x_{2t} + \beta_2 x_{2t} + \varepsilon_t \\
&= \beta_0 + \beta_1 \underbrace{(x_{1t} + x_{2t})}_{x_{1P2t}} + (\beta_2 - \beta_1) x_{2t} + \varepsilon_t \\
&= \beta_0 + \beta_1 x_{1t} - \beta_2 x_{1t} + \beta_2 x_{1t} + \beta_2 x_{2t} + \varepsilon_t \\
&= \beta_0 + (\beta_1 - \beta_2) x_{1t} + \beta_2 \underbrace{(x_{1t} + x_{2t})}_{x_{1P2t}} + \varepsilon_t
\end{aligned}
$$

Model misspecification

Assume that the variable x_{2t} is (wrongly) omitted from the model, hence we explicitly consider the conditional distribution of $(y_t \mid x_{1t})$.

$$E[y_t \mid x_{1t}] = \beta_0 + \beta_1 x_{1t} + \beta_2 E[x_{2t} \mid x_{1t}] + \underbrace{E[\varepsilon_t \mid x_{1t}]}_{0}$$

$$\begin{aligned} E[x_{2t} \mid x_{1t}] &= \alpha_0 + \alpha_1 x_{1t} \\ \alpha_0 &= \mu_2 - \frac{\sigma_{21}}{\sigma_{11}}\mu_1 \\ \alpha_1 &= \frac{\sigma_{21}}{\sigma_{11}} \end{aligned}$$

$$E[y_t \mid x_{1t}] = \underbrace{\mu_y - (\beta_1 + \beta_2\frac{\sigma_{21}}{\sigma_{11}})\mu_1}_{\delta_0} + \underbrace{(\beta_1 + \beta_2\frac{\sigma_{21}}{\sigma_{11}})}_{\delta_1} x_{1t}$$

Can we simplify the expression for δ_1 ?

$$
\begin{aligned}
\delta_1 &= \underbrace{\frac{\sigma_{1y} - \frac{\sigma_{21}}{\sigma_{22}}\sigma_{2y}}{\sigma_{11} - \frac{\sigma_{21}}{\sigma_{22}}\sigma_{12}}}_{\beta_1} + \underbrace{\frac{\sigma_{2y} - \frac{\sigma_{12}}{\sigma_{11}}\sigma_{1y}}{\sigma_{22} - \frac{\sigma_{12}}{\sigma_{11}}\sigma_{21}}}_{\beta_2} \frac{\sigma_{21}}{\sigma_{11}} \\
&= \frac{\sigma_{22}\sigma_{1y} - \sigma_{21}\sigma_{2y} + (\sigma_{11}\sigma_{2y} - \sigma_{12}\sigma_{1y})\frac{\sigma_{21}}{\sigma_{11}}}{\sigma_{11}\left(\sigma_{22} - \frac{\sigma_{12}\sigma_{21}}{\sigma_{11}}\right)} \\
&= \frac{\sigma_{1y}\left(\sigma_{22} - \sigma_{12}\frac{\sigma_{21}}{\sigma_{11}}\right)}{\sigma_{11}\left(\sigma_{22} - \frac{\sigma_{12}\sigma_{21}}{\sigma_{11}}\right)} \\
&= \frac{\sigma_{1y}}{\sigma_{11}} \quad \left(= \beta_1 + \beta_2\frac{\sigma_{21}}{\sigma_{11}}\right)
\end{aligned}
$$

- Systems of linear equations

$$y_{1t} = \pi_{10} + \pi_{11}x_{1t} + \pi_{12}x_{2t} + \omega_{1t}$$

$$y_{2t} = \pi_{20} + \pi_{21}x_{1t} + \pi_{22}x_{2t} + \omega_{2t}$$

Seemingly Unrelated Regressions (SUR)
Simultaneous models (structural form, reduced form)
Comparison of different estimators, OLS, IV, TSLS (Monte Carlo)

Main topics

- Econometric modelling - some concepts
  - Level I (Statistical model/sampling theory) model is known, parameters are known
  - Level II (Estimation course) model is known, parameters are unknown (strict exogeneity, orthogonality)
  - Level III (Econometric modeling) model is unknown, parameters are unknown (weak exogeneity)
  - Level IV (Forecasting/Policy analysis) (requires strong/super exogeneity)

- Probability distributions
  - Simultaneous density functions
  - Conditional and marginal distributions - factorization

Example 1

Example 1 *Use PcGive to investigate the potential relationship between inflation and excess demand*

Let p_t be (the log) of headline CPI, y_t denotes (the log of) output in the Norwegian mainland economy (fixed prices), and y_t^* is the log of a measure of potential output (equilibrium growth path) for the mainland economy.

Assume that the following simple relationship holds between annual rate of CPI growth, $\Delta_4 p_t$, and "output gap" measured by the (log) difference $(y_t - y_t^*)$. What is the interpretation of the "output gap" in words?

$$\Delta_4 p_t = \alpha + \beta \Delta_4 p_{t-1} + \gamma (y - y^*)_t + \varepsilon_t \tag{1}$$

assuming $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$.

Disposition

1. Read and calculate necessary variables
2. Calculate "potential output" $y_t^* = \mu_t$ by means of a detrending method
3. Calculate the "output gap" $(y_t - y_t^*)$
4. Formulate and estimate the model using OLS
5. Evaluate the model

Estimation of μ_t (detrending output)

A detrending method decomposes (the log of) real output, y_t, into a trend component, μ_t, and a cycle component, z_t, see Orphanides and van Norden (JBES,2002) for an overview over the relative merits of different detrending methods using **Final**, **RealTime** and **QuasiReal** data.

$$y_t = \mu_t + z_t \tag{2}$$

		Trend/cycle decomposition	$y_t = \mu_t + z_t$
A.			*Deterministic trends*
	LT	Linear trend	$\mu_t = \alpha + \beta t + \varepsilon_t$
	BT	Linear trend with break	$\mu_t = \alpha + \beta t + \gamma I_{t>t_0} t + \varepsilon_t$
	QT	Quadratic trend	$\mu_t = \alpha + \beta t + \gamma t^2 + \varepsilon_t$
B.			*Univariate Unobserved-Components models*
	WT	Watson(1986)	$\mu_t = \delta + \mu_{t-1} + \eta_t$ $z_t = \rho_1 z_{t-1} + \rho_2 z_{t-2} + \varepsilon_t$
	CL	Harvey(1985),Clark(1987)	$\mu_t = \delta_t + \mu_{t-1} + \eta_t$ $\delta_t = \delta_{t-1} + \nu_t$ $z_t = \rho_1 z_{t-1} + \rho_2 z_{t-2} + \varepsilon_t$
C.			*Bivariate Unobserved-Components models*
	KT	Kuttner(1994)	Watson model (WT) + $\Delta\pi_t = \xi_1 + \xi_2 \Delta y_t + \xi_3 z_{t-1} +$ MA(3)-error
	GS	Gerlach and Smets (1997)	Harvey-Clark model (CL) + $\Delta\pi_t = \phi_1 + \phi_2 z_t +$ MA(3)-error
D.			*Two-sided filter*
	HP	Hodrick-Prescott	$\mu_t = \text{argmin} \sum_{t=1}^{T} \{(y_t - \mu_t)^2 + \lambda[\Delta^2 \mu_{t+1}]\}$

Batch-commands in the example-program EO1_INFL_IA.FL:

module	module selection, e.g. PcGive (single equation modeling)
loaddata	load data from disk (in7,bn7),wk1,xls,...
algebra	data transformations
system	model specification
println	print line to output file (Results)
estsystem	estimation e.g. using OLS
store	stores "fitted" values of regression to the database

Batchfilen E01_INFL_IA.FL (for PcGive 10.0)

```
// Example E01_INFL_IA.FL
// Written by: Øyvind Eitrheim
// Content: Analysing Norwegian data,
//          data-transformations, LS-estimation
//          Inflation and the output gap

module("PcGive");                          // load PcGive module
loaddata("c:\_ects\kurs\xls\v04infl.xls"); // load data from
spreadsheet V04INFL.XLS

algebra{
y   = log(YF);
pc  = log(PC);
cpi = log(CPI);
e   = log(CPIVAL);
d4y = y-lag(y,4);
d4pc= pc-lag(pc,4);
d4cpi=cpi-lag(cpi,4);
d4e = e-lag(e,4);
t   = trend();
t2  = t^2;
```

```
t3  = t^3;
ysm=smooth_hp(y,1600,ysm);                  // calulate trend-output using the HP filter
ygapsm = y-ysm;                             //
mygapsm= movingavg(ygapsm,2,2);
dysm = diff(ysm,1);
}


system                                      // Trendmodel T1 {
    Y = y;
    Z = Constant, t;
}
println("");
println("1) Trend model T1 estimated over 1966(1)-2003(4)");
estsystem("OLS", 1966, 1, 2003, 4, 0, 0, 0);
store("fitted");
algebra{ yh1 = fitted;
dyh1 = diff(yh1,1);
ygap1 = y-yh1;
mygap1 = movingavg(ygap1,2,2);
}

setdrawwindow("Output gap plots");
draw(0,y);
draw(0,yh1);
draw(2,ygap1);
```

```
draw(2,mygap1);
draw(3,y);
draw(3,ysm);
draw(5,ygapsm);
draw(5,mygapsm);

system                                    // Trendmodell T2 {
    Y = y;
    Z = Constant, t, t2, t3;
}
println("");
println("2) Trend model T2 estimated over 1966(1)-2003(4)");
estsystem("OLS", 1966, 1, 2003, 4, 0, 0, 0);
store("fitted");
algebra{
yh2 = fitted;
dyh2 = diff(yh2,1);
ygap2 = y-yh2;
mygap2 = movingavg(ygap2,2,2);
}

draw(6,y);
draw(6,yh2);
draw(8,ygap2);
draw(8,mygap2);
draw(1,dyh1);
```

```
draw(4,dysm);
draw(7,dyh2);
show;
//break;

//
// M1:
//

algebra{
d4cpi1= lag(d4cpi,1);
d4cpip1= lag(d4cpi,-1);
ygap= ygap2;
fitted = fitted*0-9999.99;
}

system {                                          // Inflation
model M1
    Y = d4cpi;
    Z = Constant, ygap, d4cpi1;
}
println(""); println("1) Inflation model M1 estimated over 1966(1)-2003(4)");
estsystem("OLS", 1966, 1, 2003, 4, 0, 0, 0);
testsummary;
store("fitted");
algebra{ d4cpih1 = fitted; }
```

```
setdrawwindow("M1");
draw(0,d4cpi);
draw(0,d4cpih1);
show;
```

PcGive-outputfile E01_INFL_IA.OUT

```
---- GiveWin 2.10 session started at 14:00:20 on Tuesday 01 February 2005 ----

Batch file run: C:\_ects\kurs\fl10\e01_infl_ia.fl

Ox version 3.10 (Windows) (C) J.A. Doornik, 1994-2002

Ox version 3.10 (Windows) (C) J.A. Doornik, 1994-2002
---- PcGive 10.1b session started at 14:00:20 on  1-02-2005 ----
v04infl.xls loaded from c:\_ects\kurs\xls\v04infl.xls


1) Trend model T1 estimated over 1966(1)-2003(4)

EQ( 1) Modelling y by OLS (using v04infl.xls)
       The estimation sample is: 1966 (1) to 2003 (4)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant              11.4729   0.007443    1541.   0.000   0.9999
t                  0.00655364 8.440e-005     77.6   0.000   0.9757

sigma               0.0456573  RSS               0.31268848
```

```
R^2                   0.975726  F(1,150) =     6029 [0.000]**
log-likelihood          254.49  DW                      0.683 no.
of observations        152  no. of parameters           2
mean(y)                11.9743  var(y)               0.084747

   y =  + 11.47 + 0.006554*t
(SE)     (0.00744)  (8.44e-005)

fitted [1966 (1) to 2003 (4)] saved to v04infl.xls

2) Trend model T2 estimated over 1966(1)-2003(4)

*** Warning: diagonal elements of W'W are very small or very
different. Numerical accuracy is endangered, try rescaling the
data.

EQ( 2) Modelling y by OLS (using v04infl.xls)
       The estimation sample is: 1966 (1) to 2003 (4)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant              11.3597    0.01092    1040.   0.000   0.9999
t                   0.0133834  0.0006160     21.7   0.000   0.7613
t2               -9.22730e-005 9.339e-006   -9.88   0.000   0.3974
t3                3.46686e-007 4.013e-008    8.64   0.000   0.3352

sigma               0.0328267  RSS               0.159483457
```

```
R^2                   0.987619  F(3,148) =     3935 [0.000]**
log-likelihood         305.658  DW                      1.34 no.
of observations        152  no. of parameters           4
mean(y)                11.9743  var(y)              0.084747

   y =  + 11.36 + 0.01338*t - 9.227e-005*t2 + 3.467e-007*t3
(SE)     (0.0109)  (0.000616)  (9.34e-006)     (4.01e-008)

fitted [1966 (1) to 2003 (4)] saved to v04infl.xls

1) Inflation model M1 estimated over 1966(1)-2003(4)

EQ( 3) Modelling d4cpi by OLS (using v04infl.xls)
       The estimation sample is: 1967 (2) to 2003 (4)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant           0.00231858   0.001682     1.38   0.170   0.0130
ygap               0.00752636    0.02590    0.291   0.772   0.0006
d4cpi1               0.954887    0.02615     36.5   0.000   0.9025

sigma               0.0102118  RSS             0.0150164111
R^2                  0.903151  F(2,144) =    671.4 [0.000]**
log-likelihood        466.811  DW                      1.67 no.
of observations       147  no. of parameters           3
mean(d4cpi)         0.0554377  var(d4cpi)        0.00105476
```

```
d4cpi =  + 0.002319 + 0.007526*ygap + 0.9549*d4cpi1 (SE)
(0.00168)  (0.0259)        (0.0261)


AR 1-5 test:      F(5,139) =   8.3798 [0.0000]** ARCH 1-4
test:    F(4,136) =   6.5825 [0.0001]** Normality test:
Chi^2(2) =   105.78 [0.0000]** hetero test:      F(4,139) =
2.5760 [0.0402]* hetero-X test:    F(5,138) =   2.0512 [0.0753]
RESET test:       F(1,143) =  0.20052 [0.6550] fitted [1967 (2)
to 2003 (4)] saved to v04infl.xls
```

Batchfilen E01_INFL.FL (for PcGive 9.30)

```
// Example E01_INFL.FL
// Written by: Øyvind Eitrheim
// Content: Analysing Norwegian data,
//          data-transformations, LS-estimation
//          Inflation and the output gap

module("PcGive");                           // load PcGive module
loaddata("c:\_ects\kurs\xls\v04infl.xls"); // load data from spreadsheet V04INFL.XLS

algebra{
y   = log(YF);
pc  = log(PC);
cpi = log(CPI);
e   = log(CPIVAL);
d4y = y-lag(y,4);
d4pc= pc-lag(pc,4);
d4cpi=cpi-lag(cpi,4);
d4e = e-lag(e,4);
t   = trend();
t2  = t^2;
t3  = t^3;
```

```
ysm=smooth_hp(y,1600,ysm);                    // calulate trend-output using the HP filter
ygapsm = y-ysm;                               //
mygapsm= movingavg(ygapsm,2,2);
dysm = diff(ysm,1);
}


system                                        // Trendmodel T1
{
    Y = y;
    Z = Constant, t;
}
println("");
println("1) Trend model T1 estimated over 1966(1)-2004(1)");
estsystem("OLS", 1966, 1, 2004, 1, 0, 0, 0);
store("fitted");
algebra{
yh1 = fitted;
dyh1 = diff(yh1,1);
ygap1 = y-yh1;
mygap1 = movingavg(ygap1,2,2);
}

setdrawwindow("Output gap plots");
draw(0,y);
draw(0,yh1);
```

```
draw(2,ygap1);
draw(2,mygap1);
draw(3,y);
draw(3,ysm);
draw(5,ygapsm);
draw(5,mygapsm);

system                                      // Trendmodel T2
{
    Y = y;
    Z = Constant, t, t2, t3;
}
println("");
println("2) Trend model T2 estimated over 1966(1)-2004(1)");
estsystem("OLS", 1966, 1, 2004, 1, 0, 0, 0);
store("fitted");
algebra{
yh2 = fitted;
dyh2 = diff(yh2,1);
ygap2 = y-yh2;
mygap2 = movingavg(ygap2,2,2);
}

draw(6,y);
draw(6,yh2);
draw(8,ygap2);
```

```
draw(8,mygap2);
draw(1,dyh1);
draw(4,dysm);
draw(7,dyh2);
show;
//break;

//
// M0-M3:
//

algebra{
//d4cpi = D4CPI;
d4cpi1= lag(d4cpi,1);
d4cpip1= lag(d4cpi,-1);
ygap= ygap2;
fitted = fitted*0-9999.99;
}
system                                     // Inflation model M0
{
    Y = d4cpi;
    Z = Constant, ygap;
}
println("");
println("0) Inflation model M0 estimated over 1966(1)-2004(1)");
estsystem("OLS", 1966, 1, 2004, 1, 0, 0, 0);
```

```
testsummary;
store("fitted");
algebra{
d4cpih0 = fitted;
}
setdrawwindow("M0");
draw(0,d4cpi);
draw(0,d4cpih0);
show;

system
{                                          // Inflation model M1
    Y = d4cpi;
    Z = Constant, ygap, d4cpi1;
}
println("");
println("1) Inflation model M1 estimated over 1966(1)-2004(1)");
estsystem("OLS", 1966, 1, 2004, 1, 0, 0, 0);
testsummary;
store("fitted");
algebra{
d4cpih1 = fitted;
}

setdrawwindow("M1");
draw(0,d4cpi);
```

```
draw(0,d4cpih1);
show;

system                                    // Inflation model M2
{
    Y = d4cpi;
    Z = Constant, ygap, d4cpip1;
}
println("");
println("2) Inflation model M2 estimated over 1966(1)-2004(1)");
estsystem("OLS", 1966, 1, 2004, 1, 0, 0, 0);
testsummary;
store("fitted");
algebra{
d4cpih2 = fitted;
}

setdrawwindow("M2");
draw(0,d4cpi);
draw(0,d4cpih2);
show;

system                                    // Inflation model M3
{
    Y = d4cpi;
    Z = Constant, ygap, d4cpi1, d4cpip1;
```

```
}
println("");
println("3) Inflation model M3 estimated over 1966(1)-2004(1)");
estsystem("OLS", 1966, 1, 2004, 1, 0, 0, 0);
testsummary;
store("fitted");
algebra{
d4cpih3 = fitted;
}

setdrawwindow("M3");
draw(0,d4cpi);
draw(0,d4cpih3);
show;

setdrawwindow("Fitted inflation");
draw(0,d4cpi);
draw(0,d4cpih0);
draw(1,d4cpi);
draw(1,d4cpih1);
draw(2,d4cpi);
draw(2,d4cpih2);
draw(3,d4cpi);
draw(3,d4cpih3);
show;
```

PcGive-outputfilen E01-INFL.OUT

```
---- GiveWin 2.10 session started at 23:57:55 on Wednesday 09 February 2005 ----

Batch file run: C:\_ects\kurs\fl10\e01_infl.fl

Ox version 3.10 (Windows) (C) J.A. Doornik, 1994-2002

Ox version 3.10 (Windows) (C) J.A. Doornik, 1994-2002
---- PcGive 10.1b session started at 23:57:56 on  9-02-2005 ----
v04infl.xls loaded from c:\_ects\kurs\xls\v04infl.xls


1) Trend model T1 estimated over 1966(1)-2004(1)

EQ( 1) Modelling y by OLS (using v04infl.xls)
       The estimation sample is: 1966 (1) to 2003 (4)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant              11.4729   0.007443    1541.   0.000   0.9999
t                  0.00655364 8.440e-005     77.6   0.000   0.9757

sigma               0.0456573  RSS                0.31268848
```

```
R^2                 0.975726  F(1,150) =     6029 [0.000]**
log-likelihood        254.49  DW                        0.683
no. of observations      152  no. of parameters             2
mean(y)              11.9743  var(y)                 0.084747

   y =  + 11.47 + 0.006554*t
(SE)     (0.00744)  (8.44e-005)

fitted [1966 (1) to 2003 (4)] saved to v04infl.xls

2) Trend model T2 estimated over 1966(1)-2004(1)

*** Warning: diagonal elements of W'W are very small or very different.
Numerical accuracy is endangered, try rescaling the data.

EQ( 2) Modelling y by OLS (using v04infl.xls)
       The estimation sample is: 1966 (1) to 2003 (4)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant              11.3597    0.01092    1040.   0.000   0.9999
t                   0.0133834  0.0006160     21.7   0.000   0.7613
t2             -9.22730e-005 9.339e-006    -9.88   0.000   0.3974
t3              3.46686e-007 4.013e-008     8.64   0.000   0.3352

sigma               0.0328267  RSS             0.159483457
R^2                  0.987619  F(3,148) =     3935 [0.000]**
```

```
log-likelihood        305.658  DW                         1.34
no. of observations       152  no. of parameters             4
mean(y)               11.9743  var(y)                 0.084747

   y =  + 11.36 + 0.01338*t - 9.227e-005*t2 + 3.467e-007*t3
(SE)     (0.0109)  (0.000616)  (9.34e-006)     (4.01e-008)

fitted [1966 (1) to 2003 (4)] saved to v04infl.xls

0) Inflation model M0 estimated over 1966(1)-2004(1)

EQ( 3) Modelling d4cpi by OLS (using v04infl.xls)
       The estimation sample is: 1967 (1) to 2003 (4)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant            0.0553932   0.002673     20.7   0.000   0.7463
ygap                0.0792705    0.08221    0.964   0.336   0.0063

sigma               0.0325065  RSS               0.15427464
R^2                0.00632861  F(1,146) =     0.9299 [0.336]
log-likelihood        298.098  DW                        0.113
no. of observations       148  no. of parameters             2
mean(d4cpi)         0.0553402  var(d4cpi)        0.00104904

d4cpi =  + 0.05539 + 0.07927*ygap
(SE)      (0.00267)  (0.0822)
```

```
AR 1-5 test:      F(5,141) =   275.80 [0.0000]**
ARCH 1-4 test:    F(4,138) =   101.37 [0.0000]**
Normality test:   Chi^2(2) =   24.250 [0.0000]**
hetero test:      F(2,143) =  0.61287 [0.5432]
hetero-X test:    F(2,143) =  0.61287 [0.5432]
RESET test:       F(1,145) =   2.2884 [0.1325]
fitted [1967 (1) to 2003 (4)] saved to v04infl.xls

1) Inflation model M1 estimated over 1966(1)-2004(1)

EQ( 4) Modelling d4cpi by OLS (using v04infl.xls)
       The estimation sample is: 1967 (2) to 2003 (4)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant           0.00231858   0.001682     1.38   0.170   0.0130
ygap               0.00752636    0.02590    0.291   0.772   0.0006
d4cpi1               0.954887    0.02615     36.5   0.000   0.9025

sigma               0.0102118  RSS             0.0150164111
R^2                  0.903151  F(2,144) =    671.4 [0.000]**
log-likelihood        466.811  DW                      1.67
no. of observations       147  no. of parameters          3
mean(d4cpi)         0.0554377  var(d4cpi)        0.00105476
```

```
d4cpi =  + 0.002319 + 0.007526*ygap + 0.9549*d4cpi1
(SE)      (0.00168)  (0.0259)       (0.0261)


AR 1-5 test:      F(5,139) =   8.3798 [0.0000]**
ARCH 1-4 test:    F(4,136) =   6.5825 [0.0001]**
Normality test:   Chi^2(2) =   105.78 [0.0000]**
hetero test:      F(4,139) =   2.5760 [0.0402]*
hetero-X test:    F(5,138) =   2.0512 [0.0753]
RESET test:       F(1,143) =  0.20052 [0.6550]
fitted [1967 (2) to 2003 (4)] saved to v04infl.xls
(remaining observations of fitted are unchanged)

2) Inflation model M2 estimated over 1966(1)-2004(1)

EQ( 5) Modelling d4cpi by OLS (using v04infl.xls)
       The estimation sample is: 1967 (1) to 2003 (3)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant           0.00302788   0.001660     1.82   0.070   0.0226
ygap               -0.0296807    0.02574    -1.15   0.251   0.0091
d4cpip1              0.948580    0.02585     36.7   0.000   0.9034

sigma               0.0101132  RSS              0.0147279008
R^2                  0.903981  F(2,144) =    677.9 [0.000]**
log-likelihood        468.237  DW                       1.66
```

```
no. of observations       147  no. of parameters           3
mean(d4cpi)         0.0556335  var(d4cpi)         0.00104344

d4cpi =  + 0.003028 - 0.02968*ygap + 0.9486*d4cpip1
(SE)      (0.00166)  (0.0257)      (0.0259)


AR 1-5 test:      F(5,139) =   8.3073 [0.0000]**
ARCH 1-4 test:    F(4,136) =   7.8001 [0.0000]**
Normality test:   Chi^2(2) =   156.16 [0.0000]**
hetero test:      F(4,139) =   1.6491 [0.1654]
hetero-X test:    F(5,138) =   1.6683 [0.1463]
RESET test:       F(1,143) =  0.57258 [0.4505]
fitted [1967 (1) to 2003 (3)] saved to v04infl.xls
(remaining observations of fitted are unchanged)

3) Inflation model M3 estimated over 1966(1)-2004(1)

EQ( 6) Modelling d4cpi by OLS (using v04infl.xls)
       The estimation sample is: 1967 (2) to 2003 (3)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant         -0.000405206   0.001141   -0.355   0.723   0.0009
ygap               -0.0169011    0.01723   -0.981   0.328   0.0067
d4cpi1               0.500850    0.03746     13.4   0.000   0.5574
d4cpip1              0.507223    0.03723     13.6   0.000   0.5666
```

```
sigma                0.00675517  RSS              0.00647978877
R^2                    0.957695  F(3,142) =     1072 [0.000]**
log-likelihood           524.49  DW                           3
no. of observations         146  no. of parameters            4
mean(d4cpi)           0.0557337  var(d4cpi)          0.00104911

d4cpi =  - 0.0004052 - 0.0169*ygap + 0.5008*d4cpi1 + 0.5072*d4cpip1
(SE)      (0.00114)   (0.0172)      (0.0375)        (0.0372)


AR 1-5 test:      F(5,137) =   45.389 [0.0000]**
ARCH 1-4 test:    F(4,134) =   19.219 [0.0000]**
Normality test:   Chi^2(2) =   90.243 [0.0000]**
hetero test:      F(6,135) =   2.4199 [0.0297]*
hetero-X test:    F(9,132) =   18.797 [0.0000]**
RESET test:       F(1,141) =  0.25564 [0.6139]
fitted [1967 (2) to 2003 (3)] saved to v04infl.xls
(remaining observations of fitted are unchanged)
```

2. lecture

- Repetition of Example (E01_INFL.FL) : Examples using data for inflation in Norway
  - Further on batch file editing
  - Running the batch file from GiveWin and OxEdit
- Example E01-GINV (E01_GINV.FL) : Examples using data from Greene's textbook
  - Disposition (basically repetition of previous example)
- Example E02-RANN (E02_RANN.FL) : Examples using artificial data
  - the Data Generating Process (DGP)
  - Spurious regressions

Example 2

Example 2 *Using PcGive to estimate simple regression models which can help explain fixed capital formation (real investments) in the US from 1968 to 1982.* Greene's textbook, table A6.2 (on CD enclosed 4th edition), contain data for this example. The spreadsheet GREENINV.WK1 contains data for $NINV$ (real investments in nominal values), $NGNP$ (nominal GNP), CPI (headline CPI), RDIS (FED discount rate) and $INFL$ (annual inflation).

Let y_t be (the log of) real investments, g_t the (log of) real GDP, r_t the interest rate level and p_t the annual rate of inflation. t is a time trend. We assume that real investment demand, y_t, can be explained by the following simple regression model.

$$y_t = \beta_0 + \beta_1 g_t + \beta_2 r_t + \beta_3 p_t + \beta_4 t + \varepsilon_t \quad (3)$$

under the assumption that $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$.

Disposition: The PcGive-batchfile E01_GINV.FL

- Read and transform data
- Calculate y, g, r, p, t
- OLS-estimation
- Testing linear parameter restrictions

Batch-commands in the example `E01-GINV.FL`:

module	module selection, e.g. PcGive (single equation modelling)
loaddata	load data from disk (in7,bn7),wk1,xls,...
algebra	data transformations
system	model specification
println	print line to output file (Results)
estsystem	estimation e.g. using OLS
testsummary	report tests for model misspecification
testrestr	test general restrictions

Batch-file E01_GINV.FL (for PcGive 9.30)

```
// Example E01_GINV.FL (for PcGive 9.30)
// Data: Greene's textbook, table A6.1
// Written by: Øyvind Eitrheim
// Innhold: Analyse av realinvesteringer for USA - Innlesing av data,
//          data-transformasjoner, OLS-estimering

module("PcGive");                                     // load the PcGive module
loaddata("n:\eitrheim\greeninv.wk1");                 // load data from the spreadsheet GREENINV.WK1

algebra{
y = NINV/(10*CPI);
g = NGNP/(10*CPI);
r = RDIS;
p = INFL;
t = trend();
}

system                                                 // M0
{
    Y = y;
    Z = Constant, g, r, p, t;
```

```
}
println("");
println("0) General model M0 estimated over 1968-1982");
estsystem("OLS", 1968, 1, 1982, 1, 0, 0, 0);
testsummary;
testrestr
{
&2 = 0;
&3 = 0;
}
testrestr
{
&1 = 0;
&2 = 0;
&3 = 0;
}
store("Fitted");
algebra{
yh0  = Fitted;
yer0 = y-yh0;
}

system
{
    Y = y;
    Z = Constant, g, t;
```

```
}
println("");
println("1) Model M1 excluding R and P, estimated over 1968-1982");
estsystem("OLS", 1968, 1, 1982, 1, 0, 0, 0);
testsummary;
store("Fitted");
algebra{
yh1  = Fitted;
yer1 = y-yh1;
}

system
{
    Y = y;
    Z = Constant, t;
}
println("");
println("2) Model M2 excluding R, P and G, estimated over 1968-1982");
estsystem("OLS", 1968, 1, 1982, 1, 0, 0, 0);
testsummary;
store("Fitted");
algebra{
yh2  = Fitted;
yer2 = y-yh2;
}
```

PcGive-output E01_GINV.OUT

```
Batch file run: C:\kurs02\fl930\e01_ginv.fl
greeninv.wk1 loaded from c:\kurs02\wk1\greeninv.wk1


0) General model M0 estimated over 1968-1982

EQ( 1) Modelling y by OLS  (using greeninv.wk1)
The present sample is:  1968 to 1982

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant        -0.50907     0.053935   -9.439  0.0000  0.8991   0.08
g                0.67030     0.053799   12.459  0.0000  0.9395   0.07
r             -0.0024281    0.0011931   -2.035  0.0692  0.2929   0.05
p            6.4025e-005    0.0013187    0.049  0.9622  0.0002   0.06
t              -0.016590    0.0019294   -8.598  0.0000  0.8809   0.06

R^2 = 0.973497  F(4,10) = 91.83 [0.0000]  \sigma = 0.00657126  DW = 1.96
RSS = 0.0004318146622 for 5 variables and 15 observations

Instability tests, variance: 0.0750805    joint: 0.774689
Information Criteria:
```

```
SC = -9.55288  HQ = -9.79141  FPE=5.75753e-005  AIC = -9.7889

AR 1- 1 F( 1,  9) =   0.099675 [0.7594]
ARCH 1  F( 1,  8) =    0.12468 [0.7331]
Normality Chi^2(2)=    0.63596 [0.7276]
RESET   F( 1,  9) =    0.03507 [0.8556]


Wald test for general restrictions
GenRes Chi^2( 2) =     5.1257 [0.0771]


Wald test for general restrictions
GenRes Chi^2( 3) =     155.69 [0.0000] **

Fitted [1968 to 1982] saved to greeninv.wk1

1) Model M1 excluding R and P, estimated over 1968-1982

EQ( 2) Modelling y by OLS  (using greeninv.wk1)
The present sample is:  1968 to 1982

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant        -0.50022     0.060333   -8.291  0.0000  0.8514   0.15
g                0.65358     0.059800   10.929  0.0000  0.9087   0.15
t              -0.017213    0.0021435   -8.030  0.0000  0.8431   0.19
```

```
R^2 = 0.959913  F(2,12) = 143.67 [0.0000]  \sigma = 0.00737762  DW = 1.22
RSS = 0.0006531514725 for 3 variables and 15 observations

Instability tests, variance: 0.0727441    joint: 0.906331
Information Criteria:
SC = -9.50014  HQ = -9.64326  FPE=6.53151e-005  AIC = -9.64175

AR 1- 1 F( 1, 11) =    0.87824 [0.3688]
ARCH 1  F( 1, 10) =    0.94308 [0.3544]
Normality Chi^2(2)=     1.9367 [0.3797]
Xi^2    F( 4,  7) =    0.24428 [0.9043]
Xi*Xj   F( 5,  6) =    0.36202 [0.8577]
RESET   F( 1, 11) =   0.083011 [0.7786]

Fitted [1968 to 1982] saved to greeninv.wk1

2) Model M2 excluding R, P and G, estimated over 1968-1982

EQ( 3) Modelling y by OLS  (using greeninv.wk1)
The present sample is:  1968 to 1982

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant         0.15773     0.012747   12.374  0.0000  0.9217   0.09
t              0.0057129    0.0014020    4.075  0.0013  0.5609   0.10
```

```
R^2 = 0.56087  F(1,13) = 16.604 [0.0013]  \sigma = 0.02346  DW = 1.23
RSS = 0.00715483093 for 2 variables and 15 observations

Instability tests, variance: 0.333517    joint: 0.579843
Information Criteria:
SC = -7.28694  HQ = -7.38236  FPE=0.000623754  AIC = -7.38135

AR 1- 1 F( 1, 12) =      1.394 [0.2606]
ARCH 1  F( 1, 11) =    0.19089 [0.6706]
Normality Chi^2(2)=     0.3664 [0.8326]
Xi^2    F( 2, 10) =     1.3543 [0.3017]
Xi*Xj   F( 2, 10) =     1.3543 [0.3017]
RESET   F( 1, 12) =    0.92601 [0.3549]

Fitted [1968 to 1982] saved to greeninv.wk1
```

The PcGive batchfile E02_RANN.FL

1. This exercise is based on artificial data
2. Use the ranseed(arg) and rann() functions to generate the stochastic variables $eps1$ and $eps2$
3. Then generate $z1$ and $z2$ according to the formulae in the batch-file. What are their properties?
4. Generate y_t and x_t according to the formulae in the batch-file. What are their properties?
5. Regress y_t on x_t and a constant. What is a sensible estimate of the regression coefficient and how do you interpret this result?
6. Regress y_t on $z1_t$ and $z2_t$. What are the expected values of the OLS-estimates?
7. What will happen if x_t is heteroskedastic?
8. What happens if $z2_t$ is heteroskedastic?

Example E02_RANN (DGP formulae):

First, construct two normally distributed random variables $\varepsilon_{1t}, \varepsilon_{2t}$ with mean zero but with different variance:

$$\varepsilon_{1t} \sim \text{Niid}(0, (0.01)^2), \varepsilon_{2t} \sim \text{Niid}(0, (0.02)^2)$$

Then we define the variables z_{1t}, z_{2t} as follows:

$$z_{1t} = \varepsilon_{1t} + 0.5\varepsilon_{2t} + 0.005t + 0.01\text{rann()}$$

$$z_{2t} = \varepsilon_{1t} - 0.5\varepsilon_{2t} + 0.0025t + 0.01\text{rann()}$$

We note that z_{1t}, z_{2t} are linear functions of $\varepsilon_{1t}, \varepsilon_{2t}$, a linear trend t and a normally distributed random term generated by the function rann().

Now, construct the variables y_t, x_t as

$$y_t = 0.4z_{1t} + 0.2z_{2t} + 0.01\text{rann}()$$

$$x_t = 0.6z_{1t} - 0.2z_{2t} + 0.01\text{rann}()$$

1. Regress y_t on x_t and a constant:

$$y_t = \gamma_0 + \gamma_1 x_t + \varepsilon_t$$

   What is the expected value of the regression coefficient γ_1 and how do you interpret this result? Can you derive an algebraic expression for γ_1 ?

2. Regress y_t on $z1_t$ and $z2_t$:

$$y_t = \alpha_0 + \alpha_1 z_{1t} + \alpha_2 z_{2t} + \varepsilon_{y,t}$$

What are the expected values of the OLS-estimates?

3. What will happen if x_t is heteroskedastic?

4. What will happen if $z2_t$ is heteroskedastic?

The batchfile E02_RANN.FL (for PcGive 9.30)

```
// Eksempel E02_RANN.FL (for PcGive 9.30)
// Skrevet av: Øyvind Eitrheim
// Innhold: Analyse av MC-data

database("E01_RANN",1966,1,2000,4,4);

algebra{
ranseed(866464);
eps1 = 0.01*rann();
m1   = mean(eps1);
eps2 = 0.02*rann();
m2   = mean(eps2);
t = trend();
z1 = eps1 + 0.5*eps2  + 0.005*t + 0.01*rann();
z2 = eps1 - 0.5*eps2  + 0.0025*t + 0.01*rann();
//z2 = eps1 - 0.5*eps2  + 0.0025*t + 0.001*t*rann();

y  = 0.4*z1  + 0.2*z2 + 0.01*rann();
x  = 0.6*z1  - 0.2*z2 + 0.01*rann();
//x  = 0.6*z1  - 0.2*z2 + 0.0004*t*rann();
}
```

```
module("PcGive");
system
{
    Y = y;
    Z = Constant, x;
}

estsystem("RLS", 0, 1, 0, 1, 20, 16, 0);
testsummary;
store("Fitted");
algebra{
yxh = Fitted;
yxer = y-yxh;
}

system
{
    Y = y;
    Z = Constant, z1,z2;
}

estsystem("RLS", 0, 1, 0, 1, 20, 16, 0);
testsummary;
store("Fitted");
algebra{
```

```
yzh = Fitted;
yzer = y-yzh;
}
```

PcGive-outputfilen E02-RANN.OUT

```
---- GiveWin 1.30 session started at 21:48:18 on Tuesday 05 February 2002 ----

Batch file run: C:\kurs02\fl930\e02_rann.fl
---- PcGive 9.30 session started at 21:48:26 on Tuesday 05 February 2002 ----


EQ( 1) Modelling y by RLS  (using E01_RANN)
The present sample is:  1966 (1) to 2000 (4) less 20 forecasts
The forecast period is: 1996 (1) to 2000 (4)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant       0.0073634    0.0025862    2.847  0.0052  0.0643   0.24
x                0.95462     0.014780   64.588  0.0000  0.9725   0.13

R^2 = 0.972492  F(1,118) = 4171.6 [0.0000]  \sigma = 0.0143945  DW = 2.10
RSS = 0.02444993104 for 2 variables and 120 observations

Instability tests, variance: 0.188877    joint: 0.891662
Information Criteria:
SC = -8.41883  HQ = -8.44642  FPE=0.000210656  AIC = -8.46529
```

Analysis of 1-step forecasts

Date	Actual	Forecast	Y-Yhat	Forecast SE	t-value
1996 1	0.293857	0.300296	-0.00643916	0.0146375	-0.439909
1996 2	0.311670	0.281664	0.0300062	0.0145948	2.05596
1996 3	0.313766	0.291539	0.0222269	0.0146167	1.52065
1996 4	0.312033	0.299916	0.0121172	0.0146365	0.827872
1997 1	0.319969	0.303427	0.0165422	0.0146452	1.12953
1997 2	0.303110	0.324415	-0.0213050	0.0147010	-1.44922
1997 3	0.315783	0.325215	-0.00943187	0.0147032	-0.641483
1997 4	0.332504	0.314158	0.0183461	0.0146728	1.25035
1998 1	0.320755	0.319471	0.00128402	0.0146872	0.0874242
1998 2	0.330459	0.302683	0.0277759	0.0146433	1.89683
1998 3	0.345625	0.331982	0.0136430	0.0147228	0.926660
1998 4	0.326471	0.320949	0.00552113	0.0146913	0.375810
1999 1	0.323852	0.328521	-0.00466929	0.0147127	-0.317365
1999 2	0.338181	0.323195	0.0149854	0.0146975	1.01958
1999 3	0.341611	0.329972	0.0116390	0.0147169	0.790859
1999 4	0.323309	0.329923	-0.00661381	0.0147168	-0.449407
2000 1	0.329999	0.334632	-0.00463241	0.0147306	-0.314474
2000 2	0.336613	0.330272	0.00634059	0.0147178	0.430811
2000 3	0.340283	0.317668	0.0226148	0.0146823	1.54028
2000 4	0.354701	0.343259	0.0114419	0.0147570	0.775354

```
Tests of parameter constancy over: 1996 (1) to 2000 (4)
Forecast Chi^2(20)=     23.432 [0.2681]
Chow    F(20,118) =      1.048 [0.4135]
```

```
t(19) for a zero forecast innovation mean =       1.93
AR 1- 5 F( 5,113) =     0.8392 [0.5246]
ARCH 4  F( 4,110) =    0.89202 [0.4714]
Normality Chi^2(2)=    0.33786 [0.8446]
Xi^2    F( 2,115) =    0.58982 [0.5561]
Xi*Xj   F( 2,115) =    0.58982 [0.5561]
RESET   F( 1,117) =     7.8607 [0.0059] **

Fitted [1966 (1) to 2000 (4)] saved to E01_RANN

EQ( 2) Modelling y by RLS  (using E01_RANN)
The present sample is:  1966 (1) to 2000 (4) less 20 forecasts
The forecast period is: 1996 (1) to 2000 (4)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant       0.0029158    0.0017778    1.640  0.1037  0.0225   0.10
z1               0.39155     0.026756   14.634  0.0000  0.6467   0.22
z2               0.20121     0.053286    3.776  0.0003  0.1086   0.21

R^2 = 0.987589  F(2,117) = 4654.9 [0.0000]  \sigma = 0.00971012  DW = 1.98
RSS = 0.01103151751 for 3 variables and 120 observations

Instability tests, variance: 0.500439*   joint: 0.884119
Information Criteria:
SC = -9.1748  HQ = -9.21619  FPE=9.66436e-005  AIC = -9.24449
```

Analysis of 1-step forecasts

Date	Actual	Forecast	Y-Yhat	Forecast SE	t-value
1996 1	0.293857	0.298253	-0.00439594	0.00988385	-0.444760
1996 2	0.311670	0.307442	0.00422879	0.00993063	0.425833
1996 3	0.313766	0.301678	0.0120881	0.00987472	1.22414
1996 4	0.312033	0.307772	0.00426092	0.00989442	0.430639
1997 1	0.319969	0.313890	0.00607937	0.00993822	0.611717
1997 2	0.303110	0.309258	-0.00614819	0.00992069	-0.619734
1997 3	0.315783	0.322471	-0.00668710	0.00991623	-0.674359
1997 4	0.332504	0.329226	0.00327845	0.00992964	0.330168
1998 1	0.320755	0.322417	-0.00166210	0.00998110	-0.166525
1998 2	0.330459	0.316336	0.0141229	0.00990158	1.42633
1998 3	0.345625	0.329959	0.0156660	0.00993239	1.57727
1998 4	0.326471	0.330673	-0.00420289	0.00997518	-0.421335
1999 1	0.323852	0.315455	0.00839647	0.00989890	0.848222
1999 2	0.338181	0.332059	0.00612182	0.00993498	0.616188
1999 3	0.341611	0.337326	0.00428496	0.00995076	0.430617
1999 4	0.323309	0.328862	-0.00555341	0.00995348	-0.557937
2000 1	0.329999	0.336995	-0.00699540	0.00993976	-0.703779
2000 2	0.336613	0.337870	-0.00125699	0.00994193	-0.126433
2000 3	0.340283	0.348044	-0.00776056	0.0100468	-0.772442
2000 4	0.354701	0.344511	0.0101896	0.00995440	1.02363

```
Tests of parameter constancy over: 1996 (1) to 2000 (4)
Forecast Chi^2(20)=     12.396 [0.9018]
```

```
Chow    F(20,117) =    0.59792 [0.9075]

t(19) for a zero forecast innovation mean =      0.867
AR 1- 5 F( 5,112) =    0.70756 [0.6190]
ARCH 4  F( 4,109) =    0.99353 [0.4143]
Normality Chi^2(2)=    0.86667 [0.6483]
Xi^2    F( 4,112) =     1.2159 [0.3081]
Xi*Xj   F( 5,111) =     1.0189 [0.4101]
RESET   F( 1,116) =     3.0639 [0.0827]

Fitted [1966 (1) to 2000 (4)] saved to E01_RANN
```

3. lecture

- Repetition of Example E02-RANN (`E02_RANN.FL`) : Examples using artificial data
  - Algebraic results
  - Simulated examples with varying $\frac{\kappa_1}{\kappa_2}$
  - Residual autocorrelation
  - Residual heteroscedasticity
  - Misspecification diagnostics
- Example E01-SP01 (`E01_SP01.FL`) : Model specification and evaluation
  - Specific to general modelling
  - General to specific modelling

Example E02-RANN (DGP formulae): Hints

The model is given by

$$z_{1t} = \phi_1 + \varepsilon_{1t} + \delta_1 \varepsilon_{2t} + \kappa_1 t + \xi_{1t}$$

$$z_{2t} = \phi_2 + \varepsilon_{1t} + \delta_2 \varepsilon_{2t} + \kappa_2 t + \xi_{2t}$$

where

$$\varepsilon_{1t} \sim \text{Niid}(0, \sigma_1^2), \varepsilon_{2t} \sim \text{Niid}(0, \sigma_2^2)$$

The expressions for y_t and x_t are given by

$$y_t = \alpha_{0y} + \alpha_1 z_{1t} + \alpha_2 z_{2t} + \varepsilon_{yt}$$

$$x_t = \beta_{0x} + \beta_1 z_{1t} + \beta_2 z_{2t} + \varepsilon_{xt}$$

where

$$\varepsilon_{yt} \sim \text{Niid}(0, \sigma_y^2), \varepsilon_{xt} \sim \text{Niid}(0, \sigma_x^2)$$

Consider the model

$$y_t = \gamma_0 + \gamma_1 x_t + \varepsilon_t$$

We insert the expressions for $z_{1,t}$ and $z_{2,t}$ and rearrange terms into

$$y_t = \alpha_{0y} + \kappa_y t + \omega_{yt}$$

where $\alpha_{0y} = \alpha_0 + \alpha_1\phi_1 + \alpha_2\phi_2$, $\kappa_y = \alpha_1\kappa_1 + \alpha_2\kappa_2$ and
$\omega_{yt} = (\alpha_1 + \alpha_2)\varepsilon_{1t} + (\alpha_1\delta_1 + \alpha_2\delta_2)\varepsilon_{2t} + \alpha_1\xi_{1t} + \alpha_2\xi_{2t} + \varepsilon_{yt}$

$$x_t = \beta_x + \kappa_x t + \omega_{xt}$$

where $\beta_{0x} = \beta_0 + \beta_1\phi_1 + \beta_2\phi_2$, $\kappa_x = \beta_1\kappa_1 + \beta_2\kappa_2$ and
$\omega_{xt} = (\beta_1 + \beta_2)\varepsilon_{1t} + (\beta_1\delta_1 + \beta_2\delta_2)\varepsilon_{2t} + \beta_1\xi_{1t} + \beta_2\xi_{2t} + \varepsilon_{xt}$

We insert these terms in the equation above and take expectations of both sides

$$[\kappa_y - \gamma_1 \kappa_x]\, t = \gamma_0 - [\alpha_{0y} - \gamma_1 \beta_{0x}] + \underbrace{\varepsilon_t + \gamma_1 \omega_{xt} - \omega_{yt}}_{E[.]=0}$$

We see that this implies $\kappa_y - \gamma_1 \kappa_x = 0$, $\gamma_0 - [\alpha_{0y} - \gamma_1 \beta_{0x}] = 0$, and we can solve for γ_1 and γ_0:

$$\gamma_1 = \frac{\kappa_y}{\kappa_x} = \frac{\alpha_2 + \alpha_1 \frac{\kappa_1}{\kappa_2}}{\beta_2 + \beta_1 \frac{\kappa_1}{\kappa_2}}$$

$$\gamma_0 = \alpha_{0y} - \frac{\kappa_y}{\kappa_x} \beta_{0x}$$

Interpreting the linear regression model as conditional expectation

Let the DGP for $(y_t, x_t)'$ be given by the simultaneous Normal probability density function

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} \sim \text{Niid}\left(\begin{bmatrix} \mu_y \\ \mu_x \end{bmatrix}, \underbrace{\begin{bmatrix} \sigma_{yy} & \sigma_{yx} \\ \sigma_{xy} & \sigma_{xx} \end{bmatrix}}_{\Sigma} \right)$$

where μ_y and μ_x are the (unconditional) expected values of y_t and x_t, and Σ is the joint covariance matrix for $(y_t, x_t)'$.

Consider the conditional distribution of

$$(y_t|x_t) \sim \text{Niid}\left(\underbrace{\beta_0 + \beta_1 x_t}_{\mu_{y|x}}, \sigma^2_{y|x}\right)$$

where the conditional expectation and variance of $(y_t|x_t)$ are given by

$$\mu_{y|x} = \underbrace{\mu_y - \frac{\sigma_{xy}}{\sigma_{xx}}\mu_x}_{\beta_0} + \underbrace{\frac{\sigma_{xy}}{\sigma_{xx}}}_{\beta_1} x_t$$

and

$$\sigma^2_{y|x} = \sigma_{yy} - \sigma_{yx}\sigma^{-1}_{xx}\sigma_{xy} \le \sigma_{yy}$$

respectively, and the regression model can be written as

$$\begin{aligned} y_t &= \beta_0 + \beta_1 x_t + \varepsilon_t \\ (\varepsilon_t | x_t) &\sim \mathsf{Niid}\left(0, \sigma^2_{y|x}\right) \\ \beta_0 &= \mu_y - \beta_1 \mu_x \\ \beta_1 &= \frac{\sigma_{xy}}{\sigma_{xx}} \end{aligned} \quad (4)$$

Using matrix notation we can write

$$\underset{T\times 1}{y} = \underset{T\times 2}{[\iota \vdots x]} \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} = \underset{T\times 2}{X} \underset{2\times 1}{\beta} + \underset{T\times 1}{\varepsilon}, \qquad \varepsilon \sim \mathsf{N}(0, \sigma^2$$

$E[X'\varepsilon] = 0$; assuming independence between errors and RHS-variables (orthogonali

Method of moments suggests that population moments can be estimated by the corresponding sample moments such that the expectation and covariance

$$E(\varepsilon_t) = 0$$

$$Cov(\varepsilon_t, x_t) = 0$$

are estimated by the corresponding sample mean and covariance

$$\frac{1}{T}\sum_t \hat{\varepsilon}_t = 0$$

$$\frac{1}{T}\sum_t (\hat{\varepsilon}_t - 0)(x_t - \bar{x}) = 0$$

where $\hat{\varepsilon}_t = (y_t - \hat{\beta}_0 - \hat{\beta}_1 x_t)$.

We introduce the following notation

$$m_{xx} = \frac{1}{T}\sum_t (x_t - \bar{x})^2 = \frac{1}{T}\sum_t x_t^2 - \bar{x}^2$$

,

$$m_{xy} = \frac{1}{T}\sum_t (x_t - \bar{x})(y_t - \bar{y}) = \frac{1}{T}\sum_t x_t y_t - \bar{x}\bar{y}$$

,

$$m_{yy} = \frac{1}{T}\sum_t (y_t - \bar{y})^2 = \frac{1}{T}\sum_t y_t^2 - \bar{y}^2$$

.

Then

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$
$$\hat{\beta}_1 = \frac{m_{xy}}{m_{xx}}$$

OLS estimation entails minimizing the sum of squares $Q = \sum_t (y_t - \hat{\beta}_0 - \hat{\beta}_1 x_t)^2$ wrt β_0, β_1. The first order conditions are

$$\frac{\partial Q}{\partial \beta_0} = 0 \Rightarrow \bar{y} = \beta_0 + \beta_1 \bar{x}$$

$$\frac{\partial Q}{\partial \beta_1} = 0 \Rightarrow \frac{1}{T} \sum_t x_t y_t = \beta_0 \bar{x} + \beta_1 \frac{1}{T} \sum_t x_t^2$$

Inserting the expressions for m_{xx} and m_{xy} above an rearranging we find similar expressions for the OLS-estimators as with the method of moments.

Using matrix notation the normal equations and the OLS-estimator $\hat{\beta}$ are given by:

$$\hat{\beta} : \min_{\beta}(y - X\beta)'(y - X\beta)$$

$$\begin{aligned}
(X'X)\hat{\beta} = X'y \quad &\Rightarrow \quad \hat{\beta} = (X'X)^{-1}X'y \\
\hat{\beta} \quad &= \quad \beta + (X'X)^{-1}X'\varepsilon \\
\text{plim}\frac{1}{T}X'X = \underset{2\times 2}{Q_{XX}} \quad &, \quad \frac{X'\varepsilon}{\sqrt{T}} \underset{\sim}{a} \mathsf{N}(0, \sigma^2 Q_{XX}) \\
\sqrt{T}(\hat{\beta} - \beta) \quad &\underset{\sim}{a} \quad \mathsf{N}(0, \sigma^2 Q_{XX}^{-1} Q_{XX} Q_{XX}^{-1}) \\
&= \quad \mathsf{N}(0, \sigma^2 Q_{XX}^{-1})
\end{aligned}$$

Autocorrelation: $E[\varepsilon_t \varepsilon_{t-s}] = \gamma_s \neq 0$ What is the problem?

Case 1: y_{t-s} is not among the regressors in the model

1. OLS-estimates are *unbiased* and *consistent*
2. OLS-estimates are not efficient, other estimators have smaller variance
3. OLS-estimates of standard deviations are biased and 't-ratios' are distorted

Case 2: y_{t-s} is included among the regressors in the model

1. OLS-estimates are *inconsistent*

LM-test for autocorrelation

1. Save $\hat{\varepsilon}_t$ from the regression
2. Regress $\hat{\varepsilon}_t$ on $1, x_t, \hat{\varepsilon}_{t-s}$
3. Use $TR^2 \sim \chi^2$ or F-test

Heteroskedasticity: $\sigma^2 = f(x_t)$ What is the problem?

1. OLS-estimates are *unbiased* and *consistent*
2. OLS-estimates are not efficient, other estimators have smaller variance
3. OLS-estimates of standard deviations are biased and 't-ratios' are distorted

LM-test for heteroscedasiticity

1. Save $\hat{\varepsilon}_t$ from the regression
2. Regress $\hat{\varepsilon}_t^2$ on $1, x_t, x_t^2$
3. Use $TR^2 \sim \chi^2$ or F-test

Normality test: Let μ, σ_x^2 denote the mean and variance of x_t, and write $m_i = E[x_t - m_1]^i$, so that $\sigma_x^2 = m_2$. The (sample) skewness and kurtosis are defined as:

$$\sqrt{b_1} = m_3/m_2^{3/2}$$

and

$$b_2 = m_4/m_2^2$$

where $m_i = 1/T \sum_t (x_t - \bar{x})^i \; i = 1, \ldots, 4$ are the four first sample moments. Let $\beta_i, i = 1, \ldots, 4$ denote the corresponding theoretical moments.

A normally distributed variable will have $\sqrt{\beta_1} = 0$, $\beta_2 = 3$. It has been shown that the test observator

$$e_1 = 1/6 * T(\sqrt{b_1}^2 + 1/24 * T(b_2 - 3) \sim \chi^2(2)$$

cf. the Jarque and Bera(1987) LM-test for normality. Unfortunately e_1 has rather poor small sample properties. $\sqrt{b_1}$ and b_2 are not independently distributed, and the sample kurtosis especially approaches normality very slowly. The test reported by PcGive is based on Doornik and Hansen (1994), who employ a small sample correction, and adapt the test for the multivariate case.

Recursive OLS estimation:

1. Find $\hat{\beta}_t$ using OLS on the first $t > k$ observations:

$$\hat{\beta}_t = \left(\mathbf{X}_t^{'}\mathbf{X}_t\right)^{-1}\mathbf{X}_t^{'}\mathbf{y}_t.$$

2. Define the residuals

$$\begin{aligned}\hat{\varepsilon}_t &= \mathbf{y}_t - \mathbf{X}_t^{'}\hat{\beta}_t \\ &= \left(\mathbf{I} - \mathbf{X}_t^{'}\left(\mathbf{X}_t^{'}\mathbf{X}_t\right)^{-1}\mathbf{X}_t^{'}\right)\mathbf{y}_t\end{aligned}$$

3. Calculate the standard deviation of $\hat{\beta}_t$ as

$$s.e.\left(\hat{\beta}_t\right) = \sqrt{\frac{\hat{\varepsilon}_t^{'}\hat{\varepsilon}_t}{t-k} \times diag\left[\left(\mathbf{X}_t^{\prime}\mathbf{X}_t\right)^{-1}\right]}$$

4. Add one observation and find $\hat{\beta}_{t+1}$...

Repeating this gives a sequence of $\hat{\beta}$s and corresponding confidence intervals

$$\hat{\beta}_t \pm t_{0.025} \times s.e.\left(\hat{\beta}_t\right) \approx \hat{\beta}_t \pm 2 \times s.e.\left(\hat{\beta}_t\right).$$

Graphing these variables produces a plot which show whether or not the estimated parameters change as the sample size increases and converge to a constant.

Regression specification test:

1. The model may be misspecified
2. OLS-estimates may suffer from omitted variables bias
3. OLS-estimates may be subject to incorrect functional form

1. Add $\hat{y}_t^2 = (\hat{\beta}'\mathbf{X}_t)^2$, $\hat{y}_t^3 = (\hat{\beta}'\mathbf{X}_t)^3$ etc. to the original model (PcGive only allows squares)
2. Perform F-test for their exclusion

Model selection statistics:

- Models with too many parameters vs models with too few variables
- Problems with omitted variables bias vs overfitting models
- Trading off improvements in fit against a penalty which rises with k and T

1. Define the residuals

$$\begin{aligned}\hat{\varepsilon}_t &= \mathbf{y}_t - \mathbf{X}_t^{'}\hat{\beta}_t \\ &= \left(\mathbf{I} - \mathbf{X}_t^{'}\left(\mathbf{X}_t^{'}\mathbf{X}_t\right)^{-1}\mathbf{X}_t^{'}\right)\mathbf{y}_t\end{aligned}$$

2. Calculate the ML estimator of σ^2

$$\hat{\sigma}^2 = 1/T\sum_{t=1}^{T}\hat{\varepsilon}_t^2$$

3. Calculate the information criteria

$$
\begin{aligned}
SC &= \log\hat{\sigma}^2 + k * \log T/T \\
HQ &= \log\hat{\sigma}^2 + 2k * \log(\log T)/T \\
FPE &= (T+k)\hat{\sigma}^2/(T-k) \\
AIC &= \log\hat{\sigma}^2 + 2k/T
\end{aligned}
$$

3. lecture

- Last lecture: Examples using artificial data, spurious regressions, model diagnostics
  - Tests for residual autocorrelation
  - Tests for heteroscedasticity
  - Tests for non-normality
  - Tests for model misspecification
- Example E03-SP01 (`E03_SP01.FL`) : Model specification and evaluation
  - Specific to general modelling
  - General to specific modelling

Example E01-SP01: Model specification $Y = f(X)$

Given a sequence of observations of two variables y_t and x_t, we are interested in modelling the relationship between the two variables. We consider first the following three model specifications:

Model 1: $Y_t = \alpha + \beta X_t + \varepsilon_t$
Model 2: $Y_t = \alpha' + \beta' X_t + \gamma' X_{t-1} + \delta' Y_{t-1} + \varepsilon_t$
Model 3: $Y_t = \alpha + \beta X_t + u_t, \quad u_t = \rho u_{t-1} + \varepsilon_t$

where $\varepsilon_t \sim Niid(0, \sigma^2)$.

We can rewrite model 3 as

$$
\begin{aligned}
(Y_t - \alpha - \beta X_t) &= \rho(Y_{t-1} - \alpha - \beta X_{t-1}) + \varepsilon_t \\
Y_t &= \underbrace{(1-\rho)\alpha}_{\alpha'} + \underbrace{\beta}_{\beta'} X_t - \underbrace{\rho\beta}_{-\gamma'} X_{t-1} + \underbrace{\rho}_{\delta'} Y_{t-1} + \varepsilon_t
\end{aligned}
$$

The batchfile E03_SP01.FL (for PcGive 9.30)

```
// Example E03_SP01.FL
// Written by: Øyvind Eitrheim
// Content: Analysing quarterly data from 1980(1)-2000(1)
//          Models of Y=F(Const,X) etc.

module("PcGive");
loaddata("c:\kurs02\wk1\sp8101.wk1");

// Model assuming white noise errors
// y = x*b + e
system
{
    Y = Y;
    Z = Constant, X;
}
println("");
println("Modell 1)  Y = f(Const,X) fra SP8101, EstMet: OLS 1980(1)-2000(1)");
estsystem("RLS", 1980, 1, 2000, 1, 0, 16, 0);
testsummary;
store("Fitted");
algebra{
```

```
YH1   = Fitted;
YER1  = Y-YH1;
}

// Model assuming autocorrelated errors
// y = x*b + u
// u = rho*u(-1) + e
system
{
    Y = Y;
    Z = Constant, X;
}
println("");
println("Modell 2)  Y = f(Const,X) fra SP8101, EstMet: RALS 1980(2)-2000(1)");
estsystem("RALS", 1980, 2, 2000, 1, 0, 0);
testsummary;
store("Fitted");
algebra{
YH2   = Fitted;
YER2  = Y-YH2;
}

// Unrestricted model with 1 lag
// y = Const + d1*y(-1) + b0*x + b1*x(-1) + e
algebra{
Y1=lag(Y,1);
```

```
X1=lag(X,1);
}
system
{
    Y = Y;
    Z = Constant,Y1,X,X1;
}
println("");
println("Modell 3)  Y = f(Const,Y(-1),X,X(-1)) fra SP8101, EstMet: RLS 1980(2)-2000(1)");
estsystem("RLS", 1980, 1, 2000, 1, 0, 16, 0);
testsummary;
store("Fitted");
algebra{
YH3   = Fitted;
YER3  = Y-YH3;
}

// Model assuming autocorrelated errors
// y = Const + b1*y(-1) + a0*x + a1*x(-1) + u
// u = rho*u(-1) + e
system
{
    Y = Y;
    Z = Constant,Y1,X,X1;
}
println("");
```

```
println("Modell 4)  Y = f(Const,Y(-1),X,X(-1)) fra SP8101, EstMet: RALS 1980(3)-2000(1)");
estsystem("RALS", 1980, 3, 2000, 1, 0, 16, 0);
testsummary;
store("Fitted");
algebra{
YH4   = Fitted;
YER4  = Y-YH4;
}

// Unrestricted model with 2 lags
// y = Const + d1*y(-1) + d2*y(-2) + b0*x + b1*x(-1) + b2*x(-2) + e
algebra{
Y2=lag(Y,2);
X2=lag(X,2);
}
system
{
    Y = Y;
    Z = Constant,Y1,Y2,X,X1,X2;
}
println("");
println("Modell 5)  Y = f(Const,Y(-1),Y(-2),X,X(-1),X(-2)) fra SP8101, EstMet: RLS 1980(3)-2000(1)");
estsystem("RLS", 1980, 3, 2000, 1, 0, 16, 0);
testsummary;
store("Fitted");
algebra{
```

```
YH5   = Fitted;
YER5  = Y-YH5;
}
```

The batchfile E03_SP01.OUT (for PcGive 9.30)

```
---- GiveWin 1.30 session started at 23:00:50 on Wednesday 13 February 2002 ----

Batch file run: C:\kurs02\fl930\e03_sp01.fl
sp8101.wk1 loaded from c:\kurs02\wk1\sp8101.wk1

---- PcGive 9.30 session started at 23:00:57 on Wednesday 13 February 2002 ----


Modell 1)  Y = f(Const,X) fra SP8101, EstMet: OLS 1980(1)-2000(1)

EQ( 1) Modelling Y by RLS  (using sp8101.wk1)
The present sample is:  1980 (1) to 2000 (1)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant          23.843       7.1132    3.352  0.0012  0.1245   1.95**
X                0.57411     0.019959   28.764  0.0000  0.9128   2.30**

R^2 = 0.91284  F(1,79) = 827.38 [0.0000]  \sigma = 26.6013  DW = 0.0107
RSS = 55902.49779 for 2 variables and 81 observations

Instability tests, variance: 1.85871**  joint: 6.83725**
```

```
Information Criteria:
SC = 6.64542  HQ = 6.61002  FPE=725.099  AIC = 6.5863

AR 1- 5 F( 5, 74) =       1473 [0.0000] **
ARCH 4  F( 4, 71) =     4770.9 [0.0000] **
Normality Chi^2(2)=     6.8954 [0.0318] *
Xi^2    F( 2, 76) =     26.371 [0.0000] **
Xi*Xj   F( 2, 76) =     26.371 [0.0000] **
RESET   F( 1, 78) =     240.56 [0.0000] **

Fitted [1980 (1) to 2000 (1)] saved to sp8101.wk1

Modell 2)  Y = f(Const,X) fra SP8101, EstMet: RALS 1980(2)-2000(1)

EQ( 2) Modelling Y by RALS  (using sp8101.wk1)
The present sample is:  1980 (2) to 2000 (1)

Variable     Coefficient    Std.Error  t-value  t-prob
Constant         -6.9280       4.4952   -1.541  0.1274
X                0.79287     0.023787   33.332  0.0000
Uhat_1            1.0536    0.0054870  192.010  0.0000

\Sum y(t)^2 = 608356  \sigma = 1.37865
\Phi = 146.35158 for 2 variables and 80 observations (3 parameters)

Roots of the Error Polynomial
```

```
                 1.0536

ARCH 4  F( 4, 70) =     2.4679 [0.0526]
Normality Chi^2(2)=     1.5593 [0.4586]
Xi^2    F( 2, 75) =    0.55273 [0.5777]
Xi*Xj   F( 2, 75) =    0.55273 [0.5777]

Fitted [1980 (2) to 2000 (1)] saved to sp8101.wk1
(remaining observations of Fitted are unchanged)

Modell 3)  Y = f(Const,Y(-1),X,X(-1)) fra SP8101, EstMet: RLS 1980(2)-2000(1)

EQ( 3) Modelling Y by RLS  (using sp8101.wk1)
The present sample is:  1980 (2) to 2000 (1)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant        -0.61150      0.86663   -0.706  0.4826  0.0065   0.21
Y1                1.0579    0.0065883  160.572  0.0000  0.9971   0.24
X                0.88669     0.082762   10.714  0.0000  0.6016   0.21
X1              -0.93079     0.083660  -11.126  0.0000  0.6196   0.21

R^2 = 0.999764  F(3,76) = 1.0722e+005 [0.0000]  \sigma = 1.37508  DW = 0.933
RSS = 143.7044172 for 4 variables and 80 observations

Instability tests, variance: 0.123077    joint: 1.39266
Information Criteria:
```

```
SC = 0.804833  HQ = 0.733483  FPE=1.98539  AIC = 0.685732

AR 1- 5 F( 5, 71) =     6.0258 [0.0001] **
ARCH 4  F( 4, 68) =     1.8118 [0.1366]
Normality Chi^2(2)=     1.1521 [0.5621]
Xi^2    F( 6, 69) =     1.2464 [0.2938]
Xi*Xj   F( 9, 66) =     1.4939 [0.1687]
RESET   F( 1, 75) =     43.803 [0.0000] **

Fitted [1980 (2) to 2000 (1)] saved to sp8101.wk1
(remaining observations of Fitted are unchanged)

Modell 4)  Y = f(Const,Y(-1),X,X(-1)) fra SP8101, EstMet: RALS 1980(3)-2000(1)

EQ( 4) Modelling Y by OLS  (using sp8101.wk1)
The present sample is:  1980 (3) to 2000 (1)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant        -0.57263      0.92856   -0.617  0.5393  0.0050   0.21
Y1                1.0578    0.0067386  156.969  0.0000  0.9970   0.24
X                0.88392     0.086336   10.238  0.0000  0.5829   0.21
X1              -0.92799     0.087286  -10.632  0.0000  0.6011   0.21

R^2 = 0.99975  F(3,75) = 1.0014e+005 [0.0000]  \sigma = 1.38408  DW = 0.924
RSS = 143.6758457 for 4 variables and 79 observations
```

```
Instability tests, variance: 0.104945    joint: 1.37992
Information Criteria:
SC = 0.81935  HQ = 0.747442  FPE=2.01267  AIC = 0.699378

AR 1- 5 F( 5, 70) =     5.9496 [0.0001] **
ARCH 4  F( 4, 67) =      1.981 [0.1074]
Normality Chi^2(2)=     1.3331 [0.5135]
Xi^2    F( 6, 68) =     1.2448 [0.2948]
Xi*Xj   F( 9, 65) =     1.5869 [0.1379]
RESET   F( 1, 74) =     43.467 [0.0000] **

Fitted [1980 (3) to 2000 (1)] saved to sp8101.wk1
(remaining observations of Fitted are unchanged)

Modell 5)  Y = f(Const,Y(-1),Y(-2),X,X(-1),X(-2)) fra SP8101, EstMet: RLS 1980(3)-2000(1)

EQ( 5) Modelling Y by RLS  (using sp8101.wk1)
The present sample is:  1980 (3) to 2000 (1)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant        0.043997      0.56483    0.078  0.9381  0.0001   0.04
Y1                1.7502     0.070721   24.747  0.0000  0.8935   0.04
Y2              -0.74075     0.075292   -9.838  0.0000  0.5701   0.04
X               0.061394     0.099253    0.619  0.5381  0.0052   0.04
X1               0.11096      0.17958    0.618  0.5386  0.0052   0.04
X2              -0.18111      0.10115   -1.790  0.0775  0.0421   0.04
```

```
R^2 = 0.999915  F(5,73) = 1.7275e+005 [0.0000]  \sigma = 0.816334  DW = 1.74
RSS = 48.64733613 for 6 variables and 79 observations

Instability tests, variance: 0.0659627    joint: 0.780209
Information Criteria:
SC = -0.152994  HQ = -0.260855  FPE=0.717015  AIC = -0.332952

AR 1- 5 F( 5, 68) =    0.46668 [0.7997]
ARCH 4  F( 4, 65) =    0.55159 [0.6985]
Normality Chi^2(2)=    0.57008 [0.7520]
Xi^2    F(10, 62) =      1.041 [0.4208]
Xi*Xj   F(20, 52) =     1.1348 [0.3461]
RESET   F( 1, 72) =     4.6108 [0.0351] *

Fitted [1980 (3) to 2000 (1)] saved to sp8101.wk1
(remaining observations of Fitted are unchanged)
```

3. lecture (cont'd)

- Example 6 (`E06-SE.FL`) : Multiple regression analysis
  - Multicollinearity
  - Omitted variables
  - Residual regression
- Example 7 (`E07-RERE.FL`) : More on residual regression
  - Residual regression

Multiple regression analysis - an example

Let the DGP for (y_t, x_{1t}, x_{2t}) be given by

$$\begin{pmatrix} y_t \\ x_{1t} \\ x_{2t} \end{pmatrix} \simeq \text{Niid}\left(\begin{bmatrix} \mu_y \\ \mu_1 \\ \mu_2 \end{bmatrix}, \underbrace{\begin{bmatrix} \sigma_{yy} & \sigma_{y1} & \sigma_{y2} \\ \sigma_{1y} & \sigma_{11} & \sigma_{12} \\ \sigma_{2y} & \sigma_{21} & \sigma_{22} \end{bmatrix}}_{\Sigma} \right)$$

We consider the conditional distribution of

$$(y_t \mid x_{1t}, x_{2t}) \simeq \text{Niid}(\beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t}, \sigma^2)$$

where

$$\sigma^2 = \sigma_{yy} - (\sigma_{y1}\sigma_{y2}) \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \begin{pmatrix} \sigma_{1y} \\ \sigma_{2y} \end{pmatrix} \leq \sigma_{yy}$$

We have

$$
\begin{aligned}
\beta_0 &= \mu_y - \beta_1\mu_1 - \beta_2\mu_2 \\
\beta = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} &= \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}^{-1} \begin{pmatrix} \sigma_{1y} \\ \sigma_{2y} \end{pmatrix} \\
&= \frac{1}{\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21}} \begin{pmatrix} \sigma_{22} & -\sigma_{21} \\ -\sigma_{12} & \sigma_{11} \end{pmatrix} \begin{pmatrix} \sigma_{1y} \\ \sigma_{2y} \end{pmatrix} \\
\beta_1 &= \frac{\sigma_{1y} - \frac{\sigma_{21}}{\sigma_{22}}\sigma_{2y}}{\sigma_{11} - \frac{\sigma_{21}}{\sigma_{22}}\sigma_{12}} \\
\beta_2 &= \frac{\sigma_{2y} - \frac{\sigma_{12}}{\sigma_{11}}\sigma_{1y}}{\sigma_{22} - \frac{\sigma_{12}}{\sigma_{11}}\sigma_{21}}
\end{aligned}
$$

The conditional regression model (DGP)

$$
y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \varepsilon_t, \quad (\varepsilon_t \mid x_{1t}, x_{2t}) \simeq \mathsf{Niid}(0, \sigma^2)
$$

We can reparameterize the model

$$\begin{aligned}
y_t &= \beta_0 + \beta_1 x_{1t} - \beta_1 x_{2t} + \beta_1 x_{2t} + \beta_2 x_{2t} + \varepsilon_t \\
&= \beta_0 + \beta_1 \underbrace{(x_{1t} - x_{2t})}_{x_{12t}} + (\beta_1 + \beta_2) x_{2t} + \varepsilon_t \\
&= \beta_0 + \beta_1 x_{1t} - \beta_2 x_{1t} + \beta_2 x_{1t} + \beta_2 x_{2t} + \varepsilon_t \\
&= \beta_0 + (\beta_1 + \beta_2) x_{1t} + \beta_2 \underbrace{(x_{2t} - x_{1t})}_{x_{21t}} + \varepsilon_t \\
&= \beta_0 + \beta_1 x_{1t} - \beta_1 x_{2t} + \beta_1 x_{2t} + \beta_2 x_{2t} + \varepsilon_t \\
&= \beta_0 + \beta_1 \underbrace{(x_{1t} + x_{2t})}_{x_{1P2t}} + (\beta_2 - \beta_1) x_{2t} + \varepsilon_t \\
&= \beta_0 + \beta_1 x_{1t} - \beta_2 x_{1t} + \beta_2 x_{1t} + \beta_2 x_{2t} + \varepsilon_t \\
&= \beta_0 + (\beta_1 - \beta_2) x_{1t} + \beta_2 \underbrace{(x_{1t} + x_{2t})}_{x_{1P2t}} + \varepsilon_t
\end{aligned}$$

Model misspecification

Assume that the variable x_{2t} is (wrongly) omitted from the model, hence we explicitly consider the conditional distribution of $(y_t \mid x_{1t})$.

$$E[y_t \mid x_{1t}] = \beta_0 + \beta_1 x_{1t} + \beta_2 E[x_{2t} \mid x_{1t}] + \underbrace{E[\varepsilon_t \mid x_{1t}]}_{0}$$

$$\begin{aligned} E[x_{2t} \mid x_{1t}] &= \alpha_0 + \alpha_1 x_{1t} \\ \alpha_0 &= \mu_2 - \frac{\sigma_{21}}{\sigma_{11}}\mu_1 \\ \alpha_1 &= \frac{\sigma_{21}}{\sigma_{11}} \end{aligned}$$

$$E[y_t \mid x_{1t}] = \underbrace{\mu_y - (\beta_1 + \beta_2 \frac{\sigma_{21}}{\sigma_{11}})\mu_1}_{\delta_0} + \underbrace{(\beta_1 + \beta_2 \frac{\sigma_{21}}{\sigma_{11}})}_{\delta_1} x_{1t}$$

Can we simplify the expression for δ_1 ?

$$\begin{aligned}
\delta_1 &= \underbrace{\frac{\sigma_{1y} - \frac{\sigma_{21}}{\sigma_{22}}\sigma_{2y}}{\sigma_{11} - \frac{\sigma_{21}}{\sigma_{22}}\sigma_{12}}}_{\beta_1} + \underbrace{\frac{\sigma_{2y} - \frac{\sigma_{12}}{\sigma_{11}}\sigma_{1y}}{\sigma_{22} - \frac{\sigma_{12}}{\sigma_{11}}\sigma_{21}}}_{\beta_2} \frac{\sigma_{21}}{\sigma_{11}} \\
&= \frac{\sigma_{22}\sigma_{1y} - \sigma_{21}\sigma_{2y} + \left(\sigma_{11}\sigma_{2y} - \sigma_{12}\sigma_{1y}\right)\frac{\sigma_{21}}{\sigma_{11}}}{\sigma_{11}\left(\sigma_{22} - \frac{\sigma_{12}\sigma_{21}}{\sigma_{11}}\right)} \\
&= \frac{\sigma_{1y}\left(\sigma_{22} - \sigma_{12}\frac{\sigma_{21}}{\sigma_{11}}\right)}{\sigma_{11}\left(\sigma_{22} - \frac{\sigma_{12}\sigma_{21}}{\sigma_{11}}\right)} \\
&= \frac{\sigma_{1y}}{\sigma_{11}} \quad \left(= \beta_1 + \beta_2\frac{\sigma_{21}}{\sigma_{11}}\right)
\end{aligned}$$

Residual regression

Consider the following regression model where we have removed (the linear) effect from the variable z_t on resp. y_t and x_{1t}.

$$\begin{aligned} y_t &= \beta_0 + \beta_1 x_{1t} + \beta_2 z_t + \varepsilon_t \\ y_t - b_{yz} z_t &= \beta_0 + \beta_1 x_{1t} + \beta_2 z_t - b_{yz} z_t + \beta_1 b_{1z} z_t - \beta_1 b_{1z} z_t + \varepsilon_t \\ [y_t - b_{yz} z_t] &= \beta_0 + \beta_1 [x_{1t} - b_{1z} z_t] + \underbrace{(\beta_2 + \beta_1 \frac{m_{1z}}{m_{zz}} - \frac{m_{yz}}{m_{zz}})}_{0} z_t + \varepsilon_t \end{aligned}$$

m_{ij} are the empirical moments between variable i og variable j.

Note that the final parenthesis on the right hand side of the equation will equal zero. We can find β_1 by applying `OLS` in two auxilliary regressions where we calculate $b_{yz} = m_{yz}/m_{zz}$ and $b_{1z} = m_{1z}/m_{zz}$ in regressions of resp. y_t and x_{1t} on z_t. Then we calculate the *residuals* in the brackets in the equation above and finally we find the `OLS` estimate of β_1 by $\hat{\beta}_1 = \frac{m_{y1.z}}{m_{11.z}}$, where $m_{ij.z}$ are the empirical moments between variable i and variable j where we have corrected for the effect from z_t.

Compare the results above with those from applying OLS on the model:

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_{zt} z_t + \varepsilon_t$$

The batchfile E06_SE.FL (for PcGive 9.30)

```
// Example E06_SE.FL
// Written by: Øyvind Eitrheim
// Content: Analysing any of the datasets SE10001, SE10002 or SE10003
//          Each dataset contains 100 observations of Y, X1 and X2

module("PcGive");                                   // laster PcGive modulen
loaddata("c:\kurs02\wk1\se10001.wk1");              // laster data fra regnearket SE10001.WK1

algebra{
t   = trend();
X12 = X1-X2;
}
// Correcting Y, X1 and X2 for a linear trend
system
{
    Y = Y;
    Z = Constant, t;
}
println("");
println("Correct for trend in Y:  Y = f(Const,t)");
estsystem("OLS", 1976, 1, 2000, 4, 0, 0, 0);
```

```
store("Residuals");
algebra{
YT = Residuals;
}

system
{
    Y = X1;
    Z = Constant, t;
}
println("");
println("Correct for trend in X1:  X1 = f(Const,t)");
estsystem("OLS", 1976, 1, 2000, 4, 0, 0, 0);
store("Residuals");
algebra{
XT1 = Residuals;
}

system
{
    Y = X2;
    Z = Constant, t;
}
println("");
println("Correct for trend in X2:  X2 = f(Const,t)");
estsystem("OLS", 1976, 1, 2000, 4, 0, 0, 0);
```

```
store("Residuals");
algebra{
XT2 = Residuals;
}

system
{
    Y = Y;
    Z = Constant, X1, X2;
}
println("");
println("Model 1) Y = f(C,X1,X2)");
println("         Multippel regresjon");
println("");
estsystem("OLS", 1976, 1, 2000, 4, 0, 0, 0);
testsummary;
store("Fitted");
algebra{
YH1 = Fitted;
}

system
{
    Y = Y;
    Z = Constant, X12, X2;
}
```

```
println("");
println("Model 2) Y = f(C,X12,X2)");
println("         Multippel regresjon, transformerte variable");
println("");
estsystem("OLS", 1976, 1, 2000, 4, 0, 0, 0);
testsummary;
store("Fitted");
algebra{
YH2 = Fitted;
}

system
{
    Y = Y;
    Z = Constant, X1;
}
println("");
println("Model 3) Y = f(C,X1)");
println("         Omitted variable");
println("");
estsystem("OLS", 1976, 1, 2000, 4, 0, 0, 0);
testsummary;
store("Fitted");
algebra{
YH3 = Fitted;
}
```

```
system
{
    Y = Y;
    Z = Constant, X2;
}
println("");
println("Model 4) Y = f(C,X2)");
println("         Omitted variable");
println("");
estsystem("OLS", 1976, 1, 2000, 4, 0, 0, 0);
testsummary;
store("Fitted");
algebra{
YH4 = Fitted;
}

system
{
    Y = YT;
    Z = Constant, XT1, XT2;
}
println("");
println("Model 5) YT = f(C,XT1,XT2)");
println("         Residual regression");
println("");
```

```
estsystem("OLS", 1976, 1, 2000, 4, 0, 0, 0);
testsummary;
store("Fitted");
algebra{
YH5 = Fitted;
}

system
{
    Y = Y;
    Z = Constant, X1, X2, t;
}
println("");
println("Model 6) Y = f(C,X1,X2,t)");
println("         Multiple regression");
println("");
estsystem("OLS", 1976, 1, 2000, 4, 0, 0, 0);
testsummary;
store("Fitted");
algebra{
YH6 = Fitted;
}
```

The batchfile E06_SE.OUT (for PcGive 9.30)

```
---- GiveWin 1.30 session started at 22:25:37 on Tuesday 26 February 2002 ----

Batch loaded from C:\kurs02\fl930\e06_se.fl

se10001.wk1 loaded from c:\kurs02\wk1\se10001.wk1

---- PcGive 9.30 session started at 22:25:47 on Tuesday 26 February 2002 ----


Correct for trend in Y:  Y = f(Const,t)

EQ( 1) Modelling Y by OLS  (using se10001.wk1)
The present sample is:  1976 (1) to 2000 (4)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant          1.8422      0.10793   17.069  0.0000  0.7483   0.07
t               0.029558    0.0018554   15.930  0.0000  0.7214   0.04

R^2 = 0.721414  F(1,98) = 253.78 [0.0000]  \sigma = 0.53559  DW = 1.65
RSS = 28.11191902 for 2 variables and 100 observations
```

```
Instability tests, variance: 0.0931172    joint: 0.216699
Information Criteria:
SC = -1.17687  HQ = -1.20789  FPE=0.292593  AIC = -1.22898

Residuals [1976 (1) to 2000 (4)] saved to se10001.wk1

Correct for trend in X1:  X1 = f(Const,t)

EQ( 2) Modelling X1 by OLS  (using se10001.wk1)
The present sample is:  1976 (1) to 2000 (4)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant         0.94376      0.10054    9.387  0.0000  0.4735   0.10
t               0.029969    0.0017284   17.339  0.0000  0.7542   0.12

R^2 = 0.75416  F(1,98) = 300.63 [0.0000]  \sigma = 0.498926  DW = 1.77
RSS = 24.39484865 for 2 variables and 100 observations

Instability tests, variance: 0.0898342    joint: 0.38982
Information Criteria:
SC = -1.31869  HQ = -1.34971  FPE=0.253906  AIC = -1.3708

Residuals [1976 (1) to 2000 (4)] saved to se10001.wk1

Correct for trend in X2:  X2 = f(Const,t)
```

```
EQ( 3) Modelling X2 by OLS  (using se10001.wk1)
The present sample is:  1976 (1) to 2000 (4)

Variable      Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant           1.9938     0.090982   21.914  0.0000  0.8305   0.05
t                0.031206    0.0015641   19.951  0.0000  0.8024   0.04

R^2 = 0.802439  F(1,98) = 398.05 [0.0000]  \sigma = 0.451504  DW = 1.65
RSS = 19.97783729 for 2 variables and 100 observations

Instability tests, variance: 0.383551    joint: 0.501428
Information Criteria:
SC = -1.51844  HQ = -1.54946  FPE=0.207933  AIC = -1.57055

Residuals [1976 (1) to 2000 (4)] saved to se10001.wk1

Model 1) Y = f(C,X1,X2)
         Multippel regresjon


EQ( 4) Modelling Y by OLS  (using se10001.wk1)
The present sample is:  1976 (1) to 2000 (4)

Variable      Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant          0.39667      0.18010    2.202  0.0300  0.0476   0.08
X1                0.37194     0.073839    5.037  0.0000  0.2073   0.06
```

```
X2               0.56707     0.073145    7.753  0.0000  0.3826   0.06

R^2 = 0.7722  F(2,97) = 164.41 [0.0000]  \sigma = 0.486806  DW = 1.69
RSS = 22.98710999 for 3 variables and 100 observations

Instability tests, variance: 0.0977197    joint: 0.499742
Information Criteria:
SC = -1.33208  HQ = -1.37861  FPE=0.24409  AIC = -1.41024

AR 1- 5 F( 5, 92) =    0.56473 [0.7268]
ARCH 4  F( 4, 89) =     1.1976 [0.3175]
Normality Chi^2(2)=     1.8331 [0.3999]
Xi^2    F( 4, 92) =     1.4836 [0.2136]
Xi*Xj   F( 5, 91) =      1.308 [0.2676]
RESET   F( 1, 96) =     1.3956 [0.2404]

Fitted [1976 (1) to 2000 (4)] saved to se10001.wk1

Model 2) Y = f(C,X12,X2)
         Multippel regresjon, transformerte variable


EQ( 5) Modelling Y by OLS  (using se10001.wk1)
The present sample is:  1976 (1) to 2000 (4)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
```

```
Constant          0.39667      0.18010    2.202  0.0300  0.0476   0.08
X12               0.37194     0.073839    5.037  0.0000  0.2073   0.06
X2                0.93901     0.052007   18.055  0.0000  0.7707   0.06

R^2 = 0.7722  F(2,97) = 164.41 [0.0000]  \sigma = 0.486806  DW = 1.69
RSS = 22.98710999 for 3 variables and 100 observations

Instability tests, variance: 0.0977197    joint: 0.499742
Information Criteria:
SC = -1.33208  HQ = -1.37861  FPE=0.24409  AIC = -1.41024

AR 1- 5 F( 5, 92) =    0.56473 [0.7268]
ARCH 4  F( 4, 89) =     1.1976 [0.3175]
Normality Chi^2(2)=     1.8331 [0.3999]
Xi^2    F( 4, 92) =    0.57377 [0.6823]
Xi*Xj   F( 5, 91) =      1.308 [0.2676]
RESET   F( 1, 96) =     1.3956 [0.2404]

Fitted [1976 (1) to 2000 (4)] saved to se10001.wk1

Model 3) Y = f(C,X1)
         Omitted variable


EQ( 6) Modelling Y by OLS  (using se10001.wk1)
The present sample is:  1976 (1) to 2000 (4)
```

```
Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant          1.3665      0.16406    8.329  0.0000  0.4145   0.76*
X1               0.80108     0.061875   12.947  0.0000  0.6310   0.52*

R^2 = 0.631048  F(1,98) = 167.62 [0.0000]  \sigma = 0.616364  DW = 1.75
RSS = 37.23066643 for 2 variables and 100 observations

Instability tests, variance: 0.0766767    joint: 1.18236*
Information Criteria:
SC = -0.895934  HQ = -0.92695  FPE=0.387503  AIC = -0.948037

AR 1- 5 F( 5, 93) =    0.56404 [0.7273]
ARCH 4  F( 4, 90) =    0.45511 [0.7684]
Normality Chi^2(2)=    0.76291 [0.6829]
Xi^2    F( 2, 95) =    0.94545 [0.3921]
Xi*Xj   F( 2, 95) =    0.94545 [0.3921]
RESET   F( 1, 97) =     0.4963 [0.4828]

Fitted [1976 (1) to 2000 (4)] saved to se10001.wk1

Model 4) Y = f(C,X2)
         Omitted variable


EQ( 7) Modelling Y by OLS  (using se10001.wk1)
```

```
The present sample is:  1976 (1) to 2000 (4)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant         0.32462      0.20062    1.618  0.1089  0.0260   0.37
X2               0.84327     0.054096   15.589  0.0000  0.7126   0.25

R^2 = 0.712613  F(1,98) = 243 [0.0000]  \sigma = 0.543984  DW = 1.54
RSS = 29.00000136 for 2 variables and 100 observations

Instability tests, variance: 0.158138    joint: 0.793135
Information Criteria:
SC = -1.14577  HQ = -1.17679  FPE=0.301837  AIC = -1.19787

AR 1- 5 F( 5, 93) =     1.3608 [0.2463]
ARCH 4  F( 4, 90) =    0.72036 [0.5802]
Normality Chi^2(2)=     0.5697 [0.7521]
Xi^2    F( 2, 95) =    0.65522 [0.5217]
Xi*Xj   F( 2, 95) =    0.65522 [0.5217]
RESET   F( 1, 97) =     2.2316 [0.1385]

Fitted [1976 (1) to 2000 (4)] saved to se10001.wk1

Model 5) YT = f(C,XT1,XT2)
         Residual regression
```

```
EQ( 8) Modelling YT by OLS  (using se10001.wk1)
The present sample is:  1976 (1) to 2000 (4)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant    -1.1891e-016     0.048265   -0.000  1.0000  0.0000   0.06
XT1              0.28654     0.098535    2.908  0.0045  0.0802   0.05
XT2              0.46190      0.10888    4.242  0.0001  0.1565   0.27

R^2 = 0.196196  F(2,97) = 11.838 [0.0000]  \sigma = 0.482652  DW = 1.68
RSS = 22.5964747 for 3 variables and 100 observations

Instability tests, variance: 0.0839569    joint: 0.481848
Information Criteria:
SC = -1.34922  HQ = -1.39575  FPE=0.239942  AIC = -1.42738

AR 1- 5 F( 5, 92) =    0.57954 [0.7155]
ARCH 4  F( 4, 89) =     1.1591 [0.3343]
Normality Chi^2(2)=     1.5222 [0.4671]
Xi^2    F( 4, 92) =     1.7283 [0.1504]
Xi*Xj   F( 5, 91) =     1.4885 [0.2013]
RESET   F( 1, 96) =     4.8009 [0.0309] *

Fitted [1976 (1) to 2000 (4)] saved to se10001.wk1

Model 6) Y = f(C,X1,X2,t)
         Multiple regression
```

```
EQ( 9) Modelling Y by OLS  (using se10001.wk1)
The present sample is:  1976 (1) to 2000 (4)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant         0.65085      0.26674    2.440  0.0165  0.0584   0.06
X1               0.28654     0.099047    2.893  0.0047  0.0802   0.06
X2               0.46190      0.10945    4.220  0.0001  0.1565   0.07
t              0.0065565    0.0050895    1.288  0.2008  0.0170   0.06

R^2 = 0.776072  F(3,96) = 110.9 [0.0000]  \sigma = 0.48516  DW = 1.68
RSS = 22.5964747 for 4 variables and 100 observations

Instability tests, variance: 0.0839569    joint: 0.585065
Information Criteria:
SC = -1.30317  HQ = -1.3652  FPE=0.244795  AIC = -1.40738

AR 1- 5 F( 5, 91) =    0.57387 [0.7198]
ARCH 4  F( 4, 88) =     1.1461 [0.3403]
Normality Chi^2(2)=     1.5222 [0.4671]
Xi^2    F( 6, 89) =     1.0717 [0.3856]
Xi*Xj   F( 9, 86) =     1.3573 [0.2203]
RESET   F( 1, 95) =      1.143 [0.2877]

Fitted [1976 (1) to 2000 (4)] saved to se10001.wk1
```

The batchfile E07_RERE.FL (for PcGive 9.30)

```
// Example E07_RERE.FL
// Written by: Øyvind Eitrheim
// Content: Analysing any of the datasets SE10001, SE10002 or SE10003
//          Each dataset contains 100 observations of Y, X1 and X2

module("PcGive");                                    // laster PcGive modulen
loaddata("c:\kurs02\wk1\se10001.wk1");               // laster data fra regnearket SE10001.WK1

algebra{
t    = trend();
X12  = X1-X2;
X21  = X2-X1;
X1P2 = X1+X2;
}
// Correcting (Y,X2) for X1 or (Y,X1) for X2
system
{
    Y = Y;
    Z = Constant, X1;
}
println("");
```

```
println("Correct Y for X1:  Y - f(Const,X1)");
estsystem("OLS", 1976, 1, 2000, 4, 0, 0, 0);
store("Residuals");
algebra{
YR1 = Residuals;
}

system
{
    Y = X2;
    Z = Constant, X1;
}
println("");
println("Correct X2 for X1:  X2 - f(Const,X1)");
estsystem("OLS", 1976, 1, 2000, 4, 0, 0, 0);
store("Residuals");
algebra{
XR1 = Residuals;
}

system
{
    Y = Y;
    Z = Constant, X2;
}
println("");
```

```
println("Correct Y for X2:  Y - f(Const,X2)");
estsystem("OLS", 1976, 1, 2000, 4, 0, 0, 0);
store("Residuals");
algebra{
YR2 = Residuals;
}

system
{
    Y = X1;
    Z = Constant, X2;
}
println("");
println("Correct X1 for X2:  X1 - f(Const,X2)");
estsystem("OLS", 1976, 1, 2000, 4, 0, 0, 0);
store("Residuals");
algebra{
XR2 = Residuals;
}

system
{
    Y = Y;
    Z = Constant, X1, X2;
}
println("");
```

```
println("Model 1) Y = f(C,X1,X2)");
println("         Multiple regression");
println("");
estsystem("OLS", 1976, 1, 2000, 4, 0, 0, 0);
testsummary;
store("Fitted");
algebra{
YH1 = Fitted;
}

system
{
    Y = YR1;
    Z = Constant, XR1;
}
println("");
println("Model 2) YR1 = f(C,XR1)");
println("         Residual regression, correcting for X2");
println("");
estsystem("OLS", 1976, 1, 2000, 4, 0, 0, 0);
testsummary;
store("Fitted");
algebra{
YH2 = Fitted;
}
```

```
system
{
    Y = YR2;
    Z = Constant, XR2;
}
println("");
println("Model 3) YR2 = f(C,XR2)");
println("         Residual regression, correcting for X1");
println("");
estsystem("OLS", 1976, 1, 2000, 4, 0, 0, 0);
testsummary;
store("Fitted");
algebra{
YH3 = Fitted;
}

system
{
    Y = Y;
    Z = Constant, X1,X21;
}
println("");
println("Model 4) Y = f(C,X1,X21)");
println("         Multiple regression, transformed variable");
println("");
estsystem("OLS", 1976, 1, 2000, 4, 0, 0, 0);
```

```
testsummary;
store("Fitted");
algebra{
YH4 = Fitted;
}

system
{
    Y = Y;
    Z = Constant, X12,X2;
}
println("");
println("Model 5) Y = f(C,X12,X2)");
println("         Multiple regression, transformed variable");
println("");
estsystem("OLS", 1976, 1, 2000, 4, 0, 0, 0);
testsummary;
store("Fitted");
algebra{
YH5 = Fitted;
}

system
{
    Y = Y;
    Z = Constant, X1, X1P2;
```

```
}
println("");
println("Model 6) Y = f(C,X1,X1P2)");
println("         Multiple regression, transformed variable");
println("");
estsystem("OLS", 1976, 1, 2000, 4, 0, 0, 0);
testsummary;
store("Fitted");
algebra{
YH6 = Fitted;
}

system
{
    Y = Y;
    Z = Constant, X1P2, X2;
}
println("");
println("Model 7) Y = f(C,X1P2,X2)");
println("         Multiple regression, transformed variable");
println("");
estsystem("OLS", 1976, 1, 2000, 4, 0, 0, 0);
testsummary;
store("Fitted");
algebra{
YH7 = Fitted;
```

```
}
```

The batchfile E07_RERE.OUT (for PcGive 9.30)

```
---- GiveWin 1.30 session started at 23:39:28 on Tuesday 26 February 2002 ----

Batch loaded from C:\kurs02\fl930\e07_rere.fl

se10001.wk1 loaded from c:\kurs02\wk1\se10001.wk1

---- PcGive 9.30 session started at 23:40:02 on Tuesday 26 February 2002 ----


Correct Y for X1:  Y - f(Const,X1)

EQ( 1) Modelling Y by OLS  (using se10001.wk1)
The present sample is:  1976 (1) to 2000 (4)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant          1.3665      0.16406    8.329  0.0000  0.4145   0.76*
X1               0.80108     0.061875   12.947  0.0000  0.6310   0.52*

R^2 = 0.631048  F(1,98) = 167.62 [0.0000]  \sigma = 0.616364  DW = 1.75
RSS = 37.23066643 for 2 variables and 100 observations
```

```
Instability tests, variance: 0.0766767    joint: 1.18236*
Information Criteria:
SC = -0.895934  HQ = -0.92695  FPE=0.387503  AIC = -0.948037

Residuals [1976 (1) to 2000 (4)] saved to se10001.wk1

Correct X2 for X1:  X2 - f(Const,X1)

EQ( 2) Modelling X2 by OLS  (using se10001.wk1)
The present sample is:  1976 (1) to 2000 (4)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant          1.7102      0.17894    9.557  0.0000  0.4824   1.41**
X1               0.75676     0.067490   11.213  0.0000  0.5620   1.00**

R^2 = 0.561978  F(1,98) = 125.73 [0.0000]  \sigma = 0.672294  DW = 1.54
RSS = 44.29390176 for 2 variables and 100 observations

Instability tests, variance: 0.061837    joint: 2.07881**
Information Criteria:
SC = -0.72222  HQ = -0.753236  FPE=0.461018  AIC = -0.774323

Residuals [1976 (1) to 2000 (4)] saved to se10001.wk1

Correct Y for X2:  Y - f(Const,X2)
```

```
EQ( 3) Modelling Y by OLS  (using se10001.wk1)
The present sample is:  1976 (1) to 2000 (4)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant         0.32462      0.20062    1.618  0.1089  0.0260   0.37
X2               0.84327     0.054096   15.589  0.0000  0.7126   0.25

R^2 = 0.712613  F(1,98) = 243 [0.0000]  \sigma = 0.543984  DW = 1.54
RSS = 29.00000136 for 2 variables and 100 observations

Instability tests, variance: 0.158138    joint: 0.793135
Information Criteria:
SC = -1.14577  HQ = -1.17679  FPE=0.301837  AIC = -1.19787

Residuals [1976 (1) to 2000 (4)] saved to se10001.wk1

Correct X1 for X2:  X1 - f(Const,X2)

EQ( 4) Modelling X1 by OLS  (using se10001.wk1)
The present sample is:  1976 (1) to 2000 (4)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant        -0.19370      0.24561   -0.789  0.4322  0.0063   0.90**
X2               0.74261     0.066227   11.213  0.0000  0.5620   0.89**

R^2 = 0.561978  F(1,98) = 125.73 [0.0000]  \sigma = 0.665975  DW = 1.65
```

```
RSS = 43.46519713 for 2 variables and 100 observations

Instability tests, variance: 0.205718    joint: 1.3067*
Information Criteria:
SC = -0.741106  HQ = -0.772122  FPE=0.452393  AIC = -0.79321

Residuals [1976 (1) to 2000 (4)] saved to se10001.wk1

Model 1) Y = f(C,X1,X2)
         Multiple regression


EQ( 5) Modelling Y by OLS  (using se10001.wk1)
The present sample is:  1976 (1) to 2000 (4)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant         0.39667      0.18010    2.202  0.0300  0.0476   0.08
X1               0.37194     0.073839    5.037  0.0000  0.2073   0.06
X2               0.56707     0.073145    7.753  0.0000  0.3826   0.06

R^2 = 0.7722  F(2,97) = 164.41 [0.0000]  \sigma = 0.486806  DW = 1.69
RSS = 22.98710999 for 3 variables and 100 observations

Instability tests, variance: 0.0977197    joint: 0.499742
Information Criteria:
SC = -1.33208  HQ = -1.37861  FPE=0.24409  AIC = -1.41024
```

```
AR 1- 5 F( 5, 92) =    0.56473 [0.7268]
ARCH 4  F( 4, 89) =     1.1976 [0.3175]
Normality Chi^2(2)=     1.8331 [0.3999]
Xi^2    F( 4, 92) =     1.4836 [0.2136]
Xi*Xj   F( 5, 91) =      1.308 [0.2676]
RESET   F( 1, 96) =     1.3956 [0.2404]

Fitted [1976 (1) to 2000 (4)] saved to se10001.wk1

Model 2) YR1 = f(C,XR1)
         Residual regression, correcting for X2


EQ( 6) Modelling YR1 by OLS  (using se10001.wk1)
The present sample is:  1976 (1) to 2000 (4)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant     4.1951e-016     0.048432    0.000  1.0000  0.0000   0.08
XR1              0.56707     0.072771    7.793  0.0000  0.3826   0.20

R^2 = 0.382576  F(1,98) = 60.724 [0.0000]  \sigma = 0.484316  DW = 1.69
RSS = 22.98710999 for 2 variables and 100 observations

Instability tests, variance: 0.0977197    joint: 0.384785
Information Criteria:
```

```
SC = -1.37813  HQ = -1.40915  FPE=0.239254  AIC = -1.43024

AR 1- 5 F( 5, 93) =    0.56759 [0.7246]
ARCH 4  F( 4, 90) =     1.2111 [0.3117]
Normality Chi^2(2)=     1.8331 [0.3999]
Xi^2    F( 2, 95) =    0.67385 [0.5122]
Xi*Xj   F( 2, 95) =    0.67385 [0.5122]
RESET   F( 1, 97) =     1.4918 [0.2249]

Fitted [1976 (1) to 2000 (4)] saved to se10001.wk1

Model 3) YR2 = f(C,XR2)
         Residual regression, correcting for X1


EQ( 7) Modelling YR2 by OLS  (using se10001.wk1)
The present sample is:  1976 (1) to 2000 (4)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant    -3.2461e-016     0.048432   -0.000  1.0000  0.0000   0.08
XR2              0.37194     0.073461    5.063  0.0000  0.2073   0.09

R^2 = 0.207341  F(1,98) = 25.635 [0.0000]  \sigma = 0.484316  DW = 1.69
RSS = 22.98710999 for 2 variables and 100 observations

Instability tests, variance: 0.0977197    joint: 0.274289
```

```
Information Criteria:
SC = -1.37813  HQ = -1.40915  FPE=0.239254  AIC = -1.43024

AR 1- 5 F( 5, 93) =    0.56689 [0.7251]
ARCH 4  F( 4, 90) =     1.2111 [0.3117]
Normality Chi^2(2)=     1.8331 [0.3999]
Xi^2    F( 2, 95) =   0.028779 [0.9716]
Xi*Xj   F( 2, 95) =   0.028779 [0.9716]
RESET   F( 1, 97) =    0.03771 [0.8464]

Fitted [1976 (1) to 2000 (4)] saved to se10001.wk1

Model 4) Y = f(C,X1,X21)
         Multiple regression, transformed variable


EQ( 8) Modelling Y by OLS  (using se10001.wk1)
The present sample is:  1976 (1) to 2000 (4)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant         0.39667      0.18010    2.202  0.0300  0.0476   0.08
X1               0.93901     0.052007   18.055  0.0000  0.7707   0.06
X21              0.56707     0.073145    7.753  0.0000  0.3826   0.06

R^2 = 0.7722  F(2,97) = 164.41 [0.0000]  \sigma = 0.486806  DW = 1.69
RSS = 22.98710999 for 3 variables and 100 observations
```

```
Instability tests, variance: 0.0977197    joint: 0.499742
Information Criteria:
SC = -1.33208  HQ = -1.37861  FPE=0.24409  AIC = -1.41024

AR 1- 5 F( 5, 92) =    0.56473 [0.7268]
ARCH 4  F( 4, 89) =     1.1976 [0.3175]
Normality Chi^2(2)=     1.8331 [0.3999]
Xi^2    F( 4, 92) =     1.6527 [0.1678]
Xi*Xj   F( 5, 91) =      1.308 [0.2676]
RESET   F( 1, 96) =     1.3956 [0.2404]

Fitted [1976 (1) to 2000 (4)] saved to se10001.wk1

Model 5) Y = f(C,X12,X2)
         Multiple regression, transformed variable


EQ( 9) Modelling Y by OLS  (using se10001.wk1)
The present sample is:  1976 (1) to 2000 (4)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant         0.39667      0.18010    2.202  0.0300  0.0476   0.08
X12              0.37194     0.073839    5.037  0.0000  0.2073   0.06
X2               0.93901     0.052007   18.055  0.0000  0.7707   0.06
```

```
R^2 = 0.7722  F(2,97) = 164.41 [0.0000]  \sigma = 0.486806  DW = 1.69
RSS = 22.98710999 for 3 variables and 100 observations

Instability tests, variance: 0.0977197    joint: 0.499742
Information Criteria:
SC = -1.33208  HQ = -1.37861  FPE=0.24409  AIC = -1.41024

AR 1- 5 F( 5, 92) =    0.56473 [0.7268]
ARCH 4  F( 4, 89) =     1.1976 [0.3175]
Normality Chi^2(2)=     1.8331 [0.3999]
Xi^2    F( 4, 92) =    0.57377 [0.6823]
Xi*Xj   F( 5, 91) =      1.308 [0.2676]
RESET   F( 1, 96) =     1.3956 [0.2404]

Fitted [1976 (1) to 2000 (4)] saved to se10001.wk1

Model 6) Y = f(C,X1,X1P2)
         Multiple regression, transformed variable


EQ(10) Modelling Y by OLS  (using se10001.wk1)
The present sample is:  1976 (1) to 2000 (4)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
Constant         0.39667      0.18010    2.202  0.0300  0.0476   0.08
X1              -0.19513      0.13748   -1.419  0.1590  0.0203   0.06
```

```
X1P2             0.56707     0.073145    7.753  0.0000  0.3826   0.06

R^2 = 0.7722  F(2,97) = 164.41 [0.0000]  \sigma = 0.486806  DW = 1.69
RSS = 22.98710999 for 3 variables and 100 observations

Instability tests, variance: 0.0977197    joint: 0.499742
Information Criteria:
SC = -1.33208  HQ = -1.37861  FPE=0.24409  AIC = -1.41024

AR 1- 5 F( 5, 92) =    0.56473 [0.7268]
ARCH 4  F( 4, 89) =     1.1976 [0.3175]
Normality Chi^2(2)=     1.8331 [0.3999]
Xi^2    F( 4, 92) =     1.4757 [0.2160]
Xi*Xj   F( 5, 91) =      1.308 [0.2676]
RESET   F( 1, 96) =     1.3956 [0.2404]

Fitted [1976 (1) to 2000 (4)] saved to se10001.wk1

Model 7) Y = f(C,X1P2,X2)
         Multiple regression, transformed variable


EQ(11) Modelling Y by OLS  (using se10001.wk1)
The present sample is:  1976 (1) to 2000 (4)

Variable     Coefficient    Std.Error  t-value  t-prob PartR^2 Instab
```

```
Constant          0.39667      0.18010    2.202  0.0300  0.0476   0.08
X1P2              0.37194     0.073839    5.037  0.0000  0.2073   0.06
X2                0.19513      0.13748    1.419  0.1590  0.0203   0.06

R^2 = 0.7722  F(2,97) = 164.41 [0.0000]  \sigma = 0.486806  DW = 1.69
RSS = 22.98710999 for 3 variables and 100 observations

Instability tests, variance: 0.0977197    joint: 0.499742
Information Criteria:
SC = -1.33208  HQ = -1.37861  FPE=0.24409  AIC = -1.41024

AR 1- 5 F( 5, 92) =    0.56473 [0.7268]
ARCH 4  F( 4, 89) =     1.1976 [0.3175]
Normality Chi^2(2)=     1.8331 [0.3999]
Xi^2    F( 4, 92) =      1.651 [0.1682]
Xi*Xj   F( 5, 91) =      1.308 [0.2676]
RESET   F( 1, 96) =     1.3956 [0.2404]

Fitted [1976 (1) to 2000 (4)] saved to se10001.wk1
```

4. lecture

- Systems of simultaneous equations
  - Exact identification
  - Over- and underidentification
  - Simultaneity bias
- Estimation methods
  - OLS,IV,2SLS
- Comparing OLS-,IV- and 2SLS-estimators
  - Example program E11-BFM1.FL: Exact identification
  - Example program E11-BFM2.FL: Overidentified supply relationship

Exact identification

Consider the following simple static market model in which we determine $(p_t, q_t)'$ (price and quantity) in a model consisting of a supply and a demand relationship. s_t and d_t denote supply and demand shift variables respectively. The demand shift variable d_t is excluded from the supply relationship and the supply shift variable s_t is excluded from the demand relationship.

Structural form:

$$\begin{aligned} p_t &= \alpha_{pq} q_t + \beta_{ps} s_t + v_{pt} \\ q_t &= \alpha_{qp} p_t + \beta_{qd} d_t + v_{qt} \end{aligned}$$

$$\begin{bmatrix} v_{pt} \\ v_{qt} \end{bmatrix} \simeq \text{Niid}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \underbrace{\begin{bmatrix} \sigma_{pp} & \sigma_{pq} \\ \sigma_{qp} & \sigma_{qq} \end{bmatrix}}_{\Sigma} \right)$$

Reduced form:

$$
\begin{aligned}
p_t &= \frac{\alpha_{pq}\beta_{qd}}{1-\alpha_{pq}\alpha_{qp}}d_t + \frac{\beta_{ps}}{1-\alpha_{pq}\alpha_{qp}}s_t + \frac{1}{1-\alpha_{pq}\alpha_{qp}}v_{pt} + \frac{\alpha_{pq}}{1-\alpha_{pq}\alpha_{qp}}v_{qt} \\
&= \pi_{pd}d_t + \pi_{ps}s_t + u_{pt} \\
q_t &= \frac{\beta_{qd}}{1-\alpha_{pq}\alpha_{qp}}d_t + \frac{\alpha_{qp}\beta_{ps}}{1-\alpha_{pq}\alpha_{qp}}s_t + \frac{\alpha_{qp}}{1-\alpha_{pq}\alpha_{qp}}v_{pt} + \frac{1}{1-\alpha_{pq}\alpha_{qp}}v_{qt} \\
&= \pi_{qd}d_t + \pi_{qs}s_t + u_{qt}
\end{aligned}
$$

$$
\hat{\alpha}_{pq} = \frac{\hat{\pi}_{pd}}{\hat{\pi}_{qd}}, \hat{\alpha}_{qp} = \frac{\hat{\pi}_{qs}}{\hat{\pi}_{ps}}, \hat{\beta}_{ps} = \hat{\pi}_{ps}\left[1 - \frac{\hat{\pi}_{pd}}{\hat{\pi}_{qd}}\frac{\hat{\pi}_{qs}}{\hat{\pi}_{ps}}\right], \hat{\beta}_{qd} = \hat{\pi}_{qd}\left[1 - \frac{\hat{\pi}_{pd}}{\hat{\pi}_{qd}}\frac{\hat{\pi}_{qs}}{\hat{\pi}_{ps}}\right]
$$

Over- and underidentification

In this case we consider a similar model as above, i.e. a simple static market model in which we determine $(p_t, q_t)'$ (price and quantity) using a supply and a demand relationship, but note that the supply shift variable s_t is omitted from the supply relationship but enters the demand relationship instead.

Structural form:

$$\begin{aligned} p_t &= \alpha_{pq} q_t + v_{pt} \\ q_t &= \alpha_{qp} p_t + \beta_{qs} s_t + \beta_{qd} d_t + v_{qt} \end{aligned}$$

$$\begin{bmatrix} v_{pt} \\ v_{qt} \end{bmatrix} \simeq \text{Niid}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \underbrace{\begin{bmatrix} \sigma_{pp} & \sigma_{pq} \\ \sigma_{qp} & \sigma_{qq} \end{bmatrix}}_{\Sigma} \right)$$

Reduced form:

$$\begin{aligned} p_t &= \frac{\alpha_{pq}\beta_{qd}}{1-\alpha_{pq}\alpha_{qp}}d_t + \frac{\alpha_{pq}\beta_{qs}}{1-\alpha_{pq}\alpha_{qp}}s_t + \frac{1}{1-\alpha_{pq}\alpha_{qp}}v_{pt} + \frac{\alpha_{pq}}{1-\alpha_{pq}\alpha_{qp}}v_{qt} \\ &= \pi_{pd}d_t + \pi_{ps}s_t + u_{pt} \\ q_t &= \frac{\beta_{qd}}{1-\alpha_{pq}\alpha_{qp}}d_t + \frac{\beta_{qs}}{1-\alpha_{pq}\alpha_{qp}}s_t + \frac{\alpha_{qp}}{1-\alpha_{pq}\alpha_{qp}}v_{pt} + \frac{1}{1-\alpha_{pq}\alpha_{qp}}v_{qt} \\ &= \pi_{qd}d_t + \pi_{qs}s_t + u_{qt} \end{aligned}$$

$\hat{\alpha}_{pq} = \frac{\hat{\pi}_{pd}}{\hat{\pi}_{qd}}$ or $\hat{\alpha}_{pq} = \frac{\hat{\pi}_{ps}}{\hat{\pi}_{qs}}$

Example (BFM1): Exact identification

Static market model to determine $(p_t, q_t)'$ (price and quantity) using a supply and a demand relationship.

Structural form:

$$\underbrace{\begin{bmatrix} 1 & -0.5 \\ 0.5 & 1 \end{bmatrix}}_{A} \begin{bmatrix} p \\ q \end{bmatrix}_t + \underbrace{\begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix}}_{B} \begin{bmatrix} s \\ d \end{bmatrix}_t = \begin{bmatrix} v_{pt} \\ v_{qt} \end{bmatrix}$$

$$\begin{bmatrix} v_{pt} \\ v_{qt} \end{bmatrix} \simeq \mathsf{Niid}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \underbrace{\begin{bmatrix} 1.0 & 0.9 \\ 0.9 & 1.0 \end{bmatrix}}_{\Sigma} \right)$$

Reduced form:

$$\begin{bmatrix} p \\ q \end{bmatrix}_t = \underbrace{\begin{bmatrix} 0.4 & 0.2 \\ -0.2 & 0.4 \end{bmatrix}}_{\Pi=-A^{-1}B} \begin{bmatrix} s \\ d \end{bmatrix}_t + \underbrace{\begin{bmatrix} 0.8 & 0.4 \\ -0.4 & 0.8 \end{bmatrix} \begin{bmatrix} v_p \\ v_q \end{bmatrix}_t}_{u_t}$$

$$\begin{bmatrix} u_{pt} \\ u_{qt} \end{bmatrix} \simeq \mathsf{Niid}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \underbrace{\begin{bmatrix} 1.38 & 0.43 \\ 0.43 & 0.22 \end{bmatrix}}_{\underbrace{A^{-1}\Sigma A^{-1})}_{\Omega}} \right)$$

Example (BFM2): Over- and underidentification

Static market model to determine $(p_t, q_t)'$ (price and quantity) using a supply and a demand relationship.

Structural form:

$$\underbrace{\begin{bmatrix} 1 & -0.5 \\ 0.5 & 1 \end{bmatrix}}_{A} \begin{bmatrix} p \\ q \end{bmatrix}_t + \underbrace{\begin{bmatrix} 0 & 0 \\ -1 & -0.5 \end{bmatrix}}_{B} \begin{bmatrix} s \\ d \end{bmatrix}_t = \begin{bmatrix} v_{pt} \\ v_{qt} \end{bmatrix}$$

$$\begin{bmatrix} v_{pt} \\ v_{qt} \end{bmatrix} \simeq \text{Niid}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \underbrace{\begin{bmatrix} 1.9 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}}_{\Sigma} \right)$$

Reduced form:

$$\begin{bmatrix} p \\ q \end{bmatrix}_t = \underbrace{\begin{bmatrix} 0.4 & 0.2 \\ 0.8 & 0.4 \end{bmatrix}}_{\Pi=-A^{-1}B} \begin{bmatrix} s \\ d \end{bmatrix}_t + \underbrace{\begin{bmatrix} 0.8 & 0.4 \\ -0.4 & 0.8 \end{bmatrix} \begin{bmatrix} v_p \\ v_q \end{bmatrix}_t}_{u_t}$$

$$\begin{bmatrix} v_{pt} \\ v_{qt} \end{bmatrix} \simeq \text{Niid}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \underbrace{\begin{bmatrix} 1.38 & -0.37 \\ -0.37 & 0.4 \end{bmatrix}}_{\underbrace{A^{-1}\Sigma A^{-1})}_{\Omega}} \right)$$

DGP-examples: BFM1/BFM2-distributions

$$\varepsilon_t \simeq \text{Niid}(0, \underbrace{\begin{pmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{pmatrix}}_{\Lambda}),$$

$$\Pi = \Lambda_{22}^{-1}\Lambda_{21},\ \Sigma = \Lambda_{11} - \Lambda_{12}\Lambda_{22}^{-1}\Lambda_{21}:$$

BFM1:

$$\begin{bmatrix} \varepsilon_{pt} \\ \varepsilon_{qt} \\ \varepsilon_{st} \\ \varepsilon_{dt} \end{bmatrix} \simeq \text{Niid}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \underbrace{\begin{bmatrix} 1.78 & 0.43 & 0.80 & 0.40 \\ 0.43 & 0.62 & -0.40 & 0.80 \\ 0.80 & -0.40 & 2 & 0 \\ 0.4 & 0.8 & 0 & 2 \end{bmatrix}}_{\Lambda} \right)$$

BFM2:

$$\begin{bmatrix} \varepsilon_{pt} \\ \varepsilon_{qt} \\ \varepsilon_{st} \\ \varepsilon_{dt} \end{bmatrix} \simeq \text{Niid}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \underbrace{\begin{bmatrix} 1.78 & 0.43 & 0.80 & 0.40 \\ 0.43 & 2 & 1.6 & 0.80 \\ 0.80 & 1.6 & 2 & 0 \\ 0.4 & 0.8 & 0 & 2 \end{bmatrix}}_{\Lambda} \right)$$

Some linear regression models

Correctly specified supply relationship (SF), and price equation (RF)

$$
\begin{aligned}
p_t &= \alpha_{pq} q_t + \beta_{ps} s_t \quad &+ v_{pt} \\
p_t &= \pi_{ps} s_t + \pi_{pd} d_t \quad &+ u_{pt}
\end{aligned}
$$

If we insert the true parameter values from the DGP we obtain:

$$
\begin{aligned}
p_t &= 0.5 q_t + 0.5 s_t \quad &+ v_{pt} \\
p_t &= 0.4 s_t + 0.2 d_t \quad &+ u_{pt}
\end{aligned}
$$

Alternative regression models

$$
\begin{aligned}
p_t &= \gamma_{pq} q_t + \gamma_{ps} s_t && + \omega_t \\
p_t &= \gamma_{pq} q_t && + \omega_t \\
p_t &= \gamma_{ps} s_t && + \omega_t \\
p_t &= \gamma_{pd} d_t && + \omega_t
\end{aligned}
$$

Suppose the right hand side variables in these equations can be correctly considered as exogenous variables. What will be the corresponding true values of $(\gamma_{pq}, \gamma_{ps}, \gamma_{pd})$, and what will be the properties of the equation error term?

Results for $(\gamma_{pq}, \gamma_{ps}, \gamma_{pd})$

Under the assumption that the right hand side variables in the equations below are exogenous, their corresponding regression coefficients are given by the expression $\gamma_p = \Lambda^{-1}_{\text{RHS},\text{RHS}} \Lambda_{\text{RHS},p}$, where RHS index the corresponding set of right hand side variables.

BFM1:

$$
\begin{aligned}
p_t &= 1.09q_t + 0.62s_t &&+\omega_t \\
p_t &= 0.69q_t &&+\omega_t \\
p_t &= 0.4s_t &&+\omega_t \\
p_t &= 0.2d_t &&+\omega_t
\end{aligned}
$$

BFM2:

$$
\begin{aligned}
p_t &= -0.29q_t + 0.63s_t + \omega_t \\
p_t &= 0.22q_t + \omega_t \\
p_t &= 0.4s_t + \omega_t \\
p_t &= 0.2d_t + \omega_t
\end{aligned}
$$

Systems of simultaneous equations

System with n equations, n endogenous variables y_t, m exogenous variables z_t, $x_t = (y_t' z_t')'$:

Structural form:

$$\underset{n\times(n+m)}{A}\ \underset{(n+m)\times 1}{x_t} = \underset{n\times 1}{u_t}$$

$$\underset{n\times n}{B}\ \underset{n\times 1}{y_t} + \underset{n\times m}{C}\ \underset{m\times 1}{z_t} = \underset{n\times 1}{u_t}$$

$$A \underset{(n+m)\times T}{X'} = \underset{n\times T}{U'} \quad \text{where} \quad E[\frac{U'U}{T}] = \underset{n\times n}{\Omega_u}$$

Reduced form:

$$\underset{n\times 1}{y_t} = \underset{n\times m}{\Pi}\ \underset{m\times 1}{z_t} + \underset{n\times 1}{v_t}$$

$$\underset{n\times T}{Y'} = \underbrace{\underset{n\times m}{\Pi}}_{-B^{-1}C}\ \underset{m\times T}{Z'} + \underset{n\times T}{V'}$$

$$V = UB^{-1} \Rightarrow E[\frac{V'V}{T}] = \Omega_v = B^{-1}E[\frac{U'U}{T}]B^{-1} = B^{-1}\Omega_u B^{-1}$$

Estimation of an equation with exogenous explanatory variables

$$\underset{T\times 1}{y_1} = \underset{T\times m}{Z}\underset{m\times 1}{\pi_1} + \underset{T\times 1}{v_1}, \qquad v_1 \sim \mathsf{N}(0, \sigma^2 I_T)$$

$E[Z'v_1] = 0$; Independence between the residuals and the RHS-variables (orthogonality)

Normal equations and the OLS-estimator $\hat{\pi}_1$ are given by:

$$\{\hat{\pi}_1 : \min_{\pi}(y_1 - Z\pi_1)'(y_1 - Z\pi_1)\}$$

$$\begin{aligned}
(Z'Z)\hat{\pi}_1 = Z'y_1 \quad &\Rightarrow \quad \hat{\pi}_1 = (Z'Z)^{-1}Z'y_1 \\
\hat{\pi}_1 \quad &= \quad \pi_1 + (Z'Z)^{-1}Z'v_1 \\
\text{plim}\frac{1}{T}Z'Z = \underset{m\times m}{Q_{ZZ}} \quad &, \quad \frac{Z'u}{\sqrt{T}}\underset{\sim}{a}\mathsf{N}(0, \sigma^2 Q_{ZZ}) \\
\frac{Z'Z}{T}\sqrt{T}(\hat{\pi}_1 - \pi_1) \quad &= \quad \frac{Z'v_1}{\sqrt{T}} \\
\sqrt{T}(\hat{\pi}_1 - \pi_1) \quad &\underset{\sim}{a} \quad \mathsf{N}(0, \sigma_1^2 Q_{ZZ}^{-1} Q_{ZZ} Q_{ZZ}^{-1}) \\
&= \quad \mathsf{N}(0, \sigma_1^2 Q_{ZZ}^{-1})
\end{aligned}$$

Estimation of an equation which is part of a larger system of equations

We consider the first equation in the system

$$\underset{T\times 1}{y_1} = \underset{T\times n_1}{X_1}\underset{n_1\times 1}{a_1} + \underset{T\times 1}{u_1}, \qquad \underset{T\times n_1}{X_1} = (\underset{T\times(n_1-m_1)}{Y_1} \vdots \underset{T\times m_1}{Z_1})$$

$E[X_1'u_1] \neq 0$; Non-orthogonality between reiduals and the RHS-variables

Let $\underset{T\times n_1}{Z}$ be a matrix with instrumental variables $(m = n_1)$.

The normal equations and the IV-estimator $\hat{a}_1$ are given by:

$$
\begin{aligned}
(Z'X_1)\hat{a}_1 &= Z'y_1 \\
\hat{a}_1 &= (Z'X_1)^{-1}Z'y_1 \\
\text{plim}\frac{1}{T}X_1'Z &= \underset{n_1\times n_1}{Q_{XZ}} \\
\text{plim}\frac{1}{T}Z'Z &= \underset{n_1\times n_1}{Q_{ZZ}} \\
\frac{Z'u_1}{\sqrt{T}} &\underset{\sim}{a} \mathsf{N}(0,\sigma_1^2 Q_{ZZ}) \\
\sqrt{T}(\hat{a}_1 - a_1) &\underset{\sim}{a} \mathsf{N}(0,\sigma_1^2 Q_{XZ}^{-1}Q_{ZZ}Q_{ZX}^{-1}) \\
&= \mathsf{N}(0,\sigma_1^2 (Q_{XZ}Q_{ZZ}^{-1}Q_{ZX})^{-1})
\end{aligned}
$$

$m > n_1$ instruments, $\underset{T\times m}{Z}$, form new instruments $\underset{T\times m}{Z^*} = \underset{T\times m}{Z}\underset{m\times n_1}{H}$.

$$\begin{aligned}\hat{a}_1 &= ({Z^*}'X_1)^{-1}{Z^*}'y_1 \\ &= (H'Z'X_1)^{-1}H'Z'y_1\end{aligned}$$

We set

$$\begin{aligned}H = \hat{\Pi}_X &= (Z'Z)^{-1}Z'X_1 \\ \text{plim}\hat{\Pi}_X &= Q_{ZZ}^{-1}Q_{ZX} \\ Z^* = Z\hat{\Pi}_X &= Z(Z'Z)^{-1}Z'X_1 = P_Z X_1 \\ P_Z = Z(Z'Z)Z' \quad &, \quad P_Z = P_Z', \;\; P_Z = P_Z P_Z\end{aligned}$$

$$\begin{aligned}
\hat{a}_1 &= ({Z^*}' X_1)^{-1} {Z^*}' y_1 \\
&= (X_1' P_Z X_1)^{-1} X_1' P_Z y_1
\end{aligned}$$

Furthermore

$$\begin{aligned}
\text{plim}\frac{1}{T} X_1' Z^* &= \text{plim}(\frac{1}{T} X_1' P_Z X_1) \\
&= \text{plim}(\frac{1}{T} X_1' Z(Z'Z)^{-1} Z' X_1) \\
&= \text{plim}(\frac{1}{T} X_1' Z)\text{plim}(\frac{1}{T} Z'Z)^{-1}\text{plim}(\frac{1}{T} Z' X_1) \\
&= Q_{XZ} Q_{ZZ}^{-1} Q_{ZX}
\end{aligned}$$

We may write:

$$
\begin{aligned}
\sqrt{T}(\hat{a}_1 - a_1) &= \sqrt{T}(X_1' P_Z X_1)^{-1} X_1' P_z u_1 \\
&= \underbrace{(\frac{1}{T}(X_1'Z)(\frac{1}{T}Z'Z)^{-1}(\frac{1}{T}Z'X_1))^{-1}}_{(Q_{XZ}Q_{ZZ}^{-1}Q_{ZX})^{-1}} \underbrace{(\frac{1}{T}X_1'Z)(\frac{1}{T}Z'Z)^{-1}}_{Q_{XZ}Q_{ZZ}^{-1}} \quad \underbrace{\frac{Z'u_1}{\sqrt{T}}}_{\mathsf{N}(0,\sigma_1^2 Q_{ZZ})}
\end{aligned}
$$

$$
\begin{aligned}
(\frac{1}{T}(X_1'Z)(\frac{1}{T}Z'Z)^{-1}\frac{1}{T}(Z'X_1))\sqrt{T}(\hat{a}_1 - a_1) &= \frac{1}{T}(X_1'Z)(\frac{1}{T}Z'Z)^{-1}\frac{Z'u_1}{\sqrt{T}} \\
&\Downarrow \\
\sqrt{T}(\hat{a}_1 - a_1) &\underset{\sim}{a} N(0, \sigma_1^2 (Q_{XZ}Q_{ZZ}^{-1}Q_{ZX})^{-1})
\end{aligned}
$$

The expression for $\hat{a}_1$ above can be rewritten and we insert $X_1 = (Y_1 \vdots Z_1)$

$$
\begin{aligned}
\hat{a}_1 &= (X_1'Z(Z'Z)^{-1}Z'X_1)^{-1}X_1'Z(Z'Z)^{-1}Z'y_1 \\
&= \left[\begin{pmatrix} Y_1' \\ Z_1' \end{pmatrix} Z(Z'Z)^{-1}Z'(Y_1 \vdots Z_1)\right]^{-1} \begin{pmatrix} Y_1' \\ Z_1' \end{pmatrix} Z(Z'Z)^{-1}Z'y_1 \\
&= \begin{bmatrix} Y_1'P_ZY_1 & Y_1'P_ZZ_1 \\ Z_1'P_ZY_1 & Z_1'Z_1 \end{bmatrix}^{-1} \begin{pmatrix} Y_1'P_Zy_1 \\ Z_1'y_1 \end{pmatrix} \\
&= \begin{bmatrix} \underbrace{Y_1'P_Z'}_{\hat{Y}_1'}\underbrace{P_ZY_1}_{\hat{Y}_1} & \underbrace{Y_1'P_Z'}_{\hat{Y}_1'}Z_1 \\ Z_1'\underbrace{P_ZY_1}_{\hat{Y}_1} & Z_1'Z_1 \end{bmatrix}^{-1} \begin{pmatrix} \underbrace{Y_1'P_Z'}_{\hat{Y}_1'}y_1 \\ Z_1'y_1 \end{pmatrix}
\end{aligned}
$$

Note that

$$
\begin{pmatrix} Y_1' \\ Z_1' \end{pmatrix} P_Z' = X_1'P_Z' = \hat{X}_1'
$$

where $\hat{X}_1$ denote the projection of X_1 on the space spanned by the m variables in Z, such as in the model $X_1 = Z\Pi_X + V_X$:

$$\begin{aligned}\hat{\Pi}_X &= (Z'Z)^{-1}Z'X_1 \\ \hat{X}_1 &= Z\hat{\Pi}_X = Z(Z'Z)^{-1}Z'X_1 = P_Z X_1\end{aligned}$$

Since $P_Z X_1 = (P_Z Y_1 \vdots P_Z Z_1) = (\hat{Y}_1 \vdots Z_1)$ we can rewrite the expression for the IV-estimator $\hat{a}_1$ into th following form (2SLS):

$$\begin{aligned}\hat{a}_1 &= \begin{bmatrix} \hat{Y}_1'\hat{Y}_1 & \hat{Y}_1'Z_1 \\ Z_1'\hat{Y}_1 & Z_1'Z_1 \end{bmatrix}^{-1} \begin{pmatrix} \hat{Y}'y_1 \\ Z_1'y_1 \end{pmatrix} \\ &= [(\hat{Y}_1 \vdots Z_1)'(\hat{Y}_1 \vdots Z_1)]^{-1}(\hat{Y}_1 \vdots Z_1)'y_1 \\ &= (\hat{X}_1'\hat{X}_1)^{-1}\hat{X}_1'y_1\end{aligned}$$

The Hausman-test

We consider a Wald-test based on $q = \left(\hat{a}_{1,\mathsf{LS}} - \hat{a}_{1,\mathsf{IV}}\right)$

$$
\begin{aligned}
W &= q'[V_{\mathsf{IV}} - V_{\mathsf{LS}}]^{-1} q \sim \chi^2[k] \\
&= \frac{q'\left\{\left[X'P_Z X\right]^{-1} - \left(X'X\right)^{-1}\right\}^{-1} q}{s^2} \\
&= \frac{q'\left\{\left(\hat{X}'\hat{X}\right)^{-1} - \left(X'X\right)^{-1}\right\}^{-1} q}{s^2}
\end{aligned}
$$

The batchfile E10_BFM101.OUT (for PcGive 10.40)

```
---- GiveWin 2.10 session started at 16:23:22 on Monday 06 February 2006 ----

Batch file run: C:\_ects\kurs\fl10\e11_bfm1.fl

Ox version 3.10 (Windows) (C) J.A. Doornik, 1994-2001

Ox version 3.10 (Windows) (C) J.A. Doornik, 1994-2001
---- PcGive 10.40 session started at 16:23:22 on  6-02-2006 ----
dmar95.wk1 loaded from c:\_ects\kurs\wk1\dmar95.wk1

bfm101.wk1 loaded from c:\_ects\kurs\wk1\bfm101.wk1

***** Correctly specified supply curve
Supply curve:  P = f(C,Q,S), OLS

EQ( 1) Modelling P by OLS (using bfm101.wk1)
       The estimation sample is: 1 to 400

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant            0.0697819    0.04670     1.49   0.136   0.0056
Q                     1.07979    0.06487     16.6   0.000   0.4111
```

```
S                     0.620686    0.03546     17.5   0.000   0.4356

sigma                 0.931466  RSS               344.448935
R^2                   0.508698  F(2,397) =    205.5 [0.000]**
log-likelihood        -537.672  DW                       2.2
no. of observations        400  no. of parameters          3
mean(P)              0.0490669  var(P)               1.75273

AR 1-2 test:      F(2,395) =   4.2255 [0.0153]*
ARCH 1-1 test:    F(1,395) =  0.64486 [0.4224]
Normality test:   Chi^2(2) =  0.84445 [0.6556]
hetero test:      F(4,392) =  0.26135 [0.9026]
hetero-X test:    F(5,391) =  0.39600 [0.8516]
RESET test:       F(1,396) =  0.98542 [0.3215]
***** Misspecified supply curve
Supply curve:  P = f(C,Q,S), OLS

EQ( 2) Modelling P by OLS (using bfm101.wk1)
       The estimation sample is: 1 to 400

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant            0.0248621    0.06200    0.401   0.689   0.0004
Q                    0.601870    0.07823     7.69   0.000   0.1295

sigma                 1.23833  RSS               610.321728
R^2                  0.129472  F(1,398) =    59.19 [0.000]**
```

```
log-likelihood        -652.08  DW                       2.16
no. of observations       400  no. of parameters           2
mean(P)             0.0490669  var(P)                1.75273

AR 1-2 test:      F(2,396) =   2.6573 [0.0714]
ARCH 1-1 test:    F(1,396) =   2.8859 [0.0901]
Normality test:   Chi^2(2) =   1.6716 [0.4335]
hetero test:      F(2,395) =  0.59837 [0.5502]
hetero-X test:    F(2,395) =  0.59837 [0.5502]
RESET test:       F(1,397) =  0.44596 [0.5046]
***** Correctly specified price equation
Price equation (RF):  P = f(C,S,D), OLS

EQ( 3) Modelling P by OLS (using bfm101.wk1)
       The estimation sample is: 1 to 400

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant             0.103629    0.06013     1.72   0.086   0.0074
S                    0.376430    0.04132     9.11   0.000   0.1729
D                    0.146745    0.04260     3.44   0.001   0.0290

sigma                 1.19601  RSS              567.885968
R^2                      0.19  F(2,397) =    46.56 [0.000]**
log-likelihood       -637.667  DW                       2.09
no. of observations       400  no. of parameters           3
mean(P)             0.0490669  var(P)                1.75273
```

```
AR 1-2 test:      F(2,395) =   3.0443 [0.0487]*
ARCH 1-1 test:    F(1,395) =   1.3642 [0.2435]
Normality test:   Chi^2(2) =  0.48019 [0.7866]
hetero test:      F(4,392) =   1.1333 [0.3403]
hetero-X test:    F(5,391) =   1.1381 [0.3395]
RESET test:       F(1,396) = 0.094792 [0.7583]
***** Misspecified price equation
Price equation (RF):  P = f(C,S,D), OLS

EQ( 4) Modelling P by OLS (using bfm101.wk1)
       The estimation sample is: 1 to 400

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant            0.0875368    0.06077     1.44   0.150   0.0052
S                    0.372276    0.04186     8.89   0.000   0.1658

sigma                 1.21223  RSS               584.859478
R^2                   0.16579  F(1,398) =     79.1 [0.000]**
log-likelihood       -643.557  DW                      2.03
no. of observations       400  no. of parameters          2
mean(P)             0.0490669  var(P)               1.75273

AR 1-2 test:      F(2,396) =   1.8070 [0.1655]
ARCH 1-1 test:    F(1,396) =   3.0970 [0.0792]
Normality test:   Chi^2(2) =  0.72936 [0.6944]
```

```
hetero test:      F(2,395) =   2.0273 [0.1331]
hetero-X test:    F(2,395) =   2.0273 [0.1331]
RESET test:       F(1,397) =  0.20318 [0.6524]
***** Correctly specified supply curve
Supply curve:  P = f(C,Q), IVE

EQ( 5) Modelling P by IVE (using bfm101.wk1)
       The estimation sample is: 1 to 400

                  Coefficient  Std.Error  t-value  t-prob
Q              Y     0.388023    0.09950     3.90   0.000
S                    0.461542    0.04307     10.7   0.000
Constant            0.0811566    0.05298     1.53   0.126

sigma                 1.05649  RSS               443.121234
Reduced form sigma      1.196
no. of observations       400  no. of parameters          3
no. endogenous variables    2  no. of instruments         3
mean(P)             0.0490669  var(P)               1.75273
Additional instruments:
[0] = D
Testing beta = 0:   Chi^2(2) =   119.34 [0.0000]**

AR 1-2 test:      F(2,395) =   3.0621 [0.0479]*
ARCH 1-1 test:    F(1,395) =   1.6693 [0.1971]
Normality test:   Chi^2(2) =  0.71004 [0.7012]
```

```
hetero test:      F(4,392) =   5.1882 [0.0004]**
hetero-X test:    F(5,391) =   4.3269 [0.0008]**
***** Correctly specified system
System:  (P,Q) = f(C,S,D), OLS

SYS( 6) Estimating the system by OLS (using bfm101.wk1)
        The estimation sample is: 1 to 400

URF equation for: P
                  Coefficient  Std.Error  t-value  t-prob
S                    0.376430    0.04132     9.11   0.000
D                    0.146745    0.04260     3.44   0.001
Constant       U     0.103629    0.06013     1.72   0.086

sigma = 1.19601   RSS = 567.8859679

URF equation for: Q
                  Coefficient  Std.Error  t-value  t-prob
S                   -0.219346    0.01676    -13.1   0.000
D                    0.378187    0.01728     21.9   0.000
Constant       U    0.0579151    0.02440     2.37   0.018

sigma = 0.485192   RSS = 93.4582914

log-likelihood    -729.665772  -T/2log|Omega|     405.485055
|Omega|            0.13167411  log|Y'Y/T|      -0.0451642009
```

```
R^2(LR)              0.862243  R^2(LM)               0.49697
no. of observations       400  no. of parameters           6

F-test on regressors except unrestricted: F(4,792) = 335.467 [0.0000] **
F-tests on retained regressors, F(2,396) =
           S       552.144 [0.000]**          D       469.487 [0.000]**
    Constant U     2.82915 [0.060]

correlation of URF residuals (standard deviations on diagonal)
                  P            Q
P            1.1960      0.77656
Q           0.77656      0.48519
correlation between actual and fitted
            P            Q
      0.43589      0.79185

P           : Portmanteau(12): 19.394
Q           : Portmanteau(12): 16.886
P           : AR 1-2 test:      F(2,395) =   3.0443 [0.0487]*
Q           : AR 1-2 test:      F(2,395) =   1.6114 [0.2009]
P           : Normality test:   Chi^2(2) =  0.48019 [0.7866]
Q           : Normality test:   Chi^2(2) =   3.2973 [0.1923]
P           : ARCH 1-1 test:    F(1,395) =   1.3642 [0.2435]
Q           : ARCH 1-1 test:    F(1,395) =  0.11291 [0.7370]
P           : hetero test:      F(4,392) =   1.1333 [0.3403]
Q           : hetero test:      F(4,392) =   1.1031 [0.3547]
```

```
P           : hetero-X test:    F(5,391) =   1.1381 [0.3395]
Q           : hetero-X test:    F(5,391) =  0.94739 [0.4502]

Vector Portmanteau(12): 47.6138
Vector AR 1-2 test:      F(8,784) =   1.0564 [0.3919]
Vector Normality test:   Chi^2(4) =   5.9089 [0.2061]
Vector hetero test:      F(12,1032)=  0.79885 [0.6520]
Vector hetero-X test:    F(15,1074)=  0.84663 [0.6254]
***** Correctly specified model
Order condition for P not satisfied!
Order condition for Q not satisfied!

MOD( 7) Estimating the model by FIML (using bfm101.wk1)
        The estimation sample is: 1 to 400

Equation for: P
                  Coefficient  Std.Error  t-value  t-prob
Q                    0.388023    0.09950     3.90   0.000
S                    0.461542    0.04307     10.7   0.000
Constant       U    0.0811566    0.05298     1.53   0.126

sigma = 1.05649

Equation for: Q
                  Coefficient  Std.Error  t-value  t-prob
P                   -0.582701     0.1024    -5.69   0.000
```

```
D                    0.463696    0.04210     11.0   0.000
Constant       U     0.118300    0.05636     2.10   0.036

sigma = 1.11637

log-likelihood    -729.665772  -T/2log|Omega|     405.485055
no. of observations       400  no. of parameters           6
No restrictions imposed
BFGS using analytical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence

correlation of structural residuals (standard deviations on diagonal)
                  P            Q
P            1.0565      0.92495
Q           0.92495       1.1164

P           : Portmanteau(12): 19.394
Q           : Portmanteau(12): 16.886
P           : AR 1-2 test:      F(2,395) =   3.0443 [0.0487]*
Q           : AR 1-2 test:      F(2,395) =   1.6114 [0.2009]
P           : Normality test:   Chi^2(2) =  0.48019 [0.7866]
Q           : Normality test:   Chi^2(2) =   3.2973 [0.1923]
P           : ARCH 1-1 test:    F(1,395) =   1.3642 [0.2435]
Q           : ARCH 1-1 test:    F(1,395) =  0.11291 [0.7370]
P           : hetero test:      F(4,392) =   1.1333 [0.3403]
Q           : hetero test:      F(4,392) =   1.1031 [0.3547]
```

```
P           : hetero-X test:    F(5,391) =   1.1381 [0.3395]
Q           : hetero-X test:    F(5,391) =  0.94739 [0.4502]

Vector Portmanteau(12): 47.6138
Vector EGE-AR 1-2 test:  F(8,784) =   1.0564 [0.3919]
Vector Normality test:   Chi^2(4) =   5.9089 [0.2061]
Vector hetero test:      F(12,1032)=  0.79885 [0.6520]
Vector hetero-X test:    F(15,1074)=  0.84663 [0.6254]
***** Misspecified model - Instr: S,D

EQ( 8) Modelling P by IVE (using bfm101.wk1)
       The estimation sample is: 1 to 400

                  Coefficient  Std.Error  t-value  t-prob
Q              Y      1.78933     0.6563     2.73   0.007
S                    0.768913     0.1482     5.19   0.000
D                   -0.529954     0.2476    -2.14   0.033

sigma                0.755973  RSS               226.883533
Reduced form sigma      1.196
no. of observations       400  no. of parameters          3
no. endogenous variables    2  no. of instruments         3
mean(P)             0.0490669  var(P)               1.75273
Additional instruments:
[0] = Constant
Testing beta = 0:   Chi^2(2) =   233.09 [0.0000]**
```

```
AR 1-2 test:      F(2,395) =  0.84123 [0.4320]
ARCH 1-1 test:    F(1,395) =   1.4083 [0.2360]
Normality test:   Chi^2(2) =   2.9451 [0.2293]
hetero test:      F(6,390) =  0.81962 [0.5551]
hetero-X test:    F(9,387) =  0.89056 [0.5337]
***** Correctly specified system
System:  (P,Q) = f(C,S,D), OLS

SYS( 9) Estimating the system by OLS (using bfm101.wk1)
        The estimation sample is: 1 to 400

URF equation for: P
                  Coefficient  Std.Error  t-value  t-prob
S                    0.376430    0.04132     9.11   0.000
D                    0.146745    0.04260     3.44   0.001
Constant       U     0.103629    0.06013     1.72   0.086

sigma = 1.19601   RSS = 567.8859679

URF equation for: Q
                  Coefficient  Std.Error  t-value  t-prob
S                   -0.219346    0.01676    -13.1   0.000
D                    0.378187    0.01728     21.9   0.000
Constant       U    0.0579151    0.02440     2.37   0.018
```

```
sigma = 0.485192   RSS = 93.4582914

log-likelihood    -729.665772  -T/2log|Omega|     405.485055
|Omega|            0.13167411  log|Y'Y/T|      -0.0451642009
R^2(LR)              0.862243  R^2(LM)                0.49697
no. of observations       400  no. of parameters            6

F-test on regressors except unrestricted: F(4,792) = 335.467 [0.0000] **
F-tests on retained regressors, F(2,396) =
           S       552.144 [0.000]**           D       469.487 [0.000]**
    Constant U     2.82915 [0.060]

correlation of URF residuals (standard deviations on diagonal)
                 P             Q
P           1.1960      0.77656
Q          0.77656      0.48519
correlation between actual and fitted
           P            Q
      0.43589      0.79185

P           : Portmanteau(12): 19.394
Q           : Portmanteau(12): 16.886
P           : AR 1-2 test:      F(2,395) =   3.0443 [0.0487]*
Q           : AR 1-2 test:      F(2,395) =   1.6114 [0.2009]
P           : Normality test:   Chi^2(2) =  0.48019 [0.7866]
Q           : Normality test:   Chi^2(2) =   3.2973 [0.1923]
```

```
P           : ARCH 1-1 test:    F(1,395) =   1.3642 [0.2435]
Q           : ARCH 1-1 test:    F(1,395) =  0.11291 [0.7370]
P           : hetero test:      F(4,392) =   1.1333 [0.3403]
Q           : hetero test:      F(4,392) =   1.1031 [0.3547]
P           : hetero-X test:    F(5,391) =   1.1381 [0.3395]
Q           : hetero-X test:    F(5,391) =  0.94739 [0.4502]

Vector Portmanteau(12): 47.6138
Vector AR 1-2 test:      F(8,784) =   1.0564 [0.3919]
Vector Normality test:   Chi^2(4) =   5.9089 [0.2061]
Vector hetero test:      F(12,1032)=  0.79885 [0.6520]
Vector hetero-X test:    F(15,1074)=  0.84663 [0.6254]
***** Misspecified model - Instr: S

EQ(10) Modelling P by IVE (using bfm101.wk1)
       The estimation sample is: 1 to 400

                  Coefficient  Std.Error  t-value  t-prob
Q              Y      1.78933     0.6563     2.73   0.007
S                    0.768913     0.1482     5.19   0.000
D                   -0.529954     0.2476    -2.14   0.033

sigma                0.755973  RSS               226.883533
Reduced form sigma      1.196
no. of observations       400  no. of parameters          3
no. endogenous variables    2  no. of instruments         3
```

```
mean(P)             0.0490669  var(P)                1.75273
Additional instruments:
[0] = Constant
Testing beta = 0:   Chi^2(2) =   233.09 [0.0000]**

AR 1-2 test:       F(2,395) =  0.84123 [0.4320]
ARCH 1-1 test:     F(1,395) =   1.4083 [0.2360]
Normality test:    Chi^2(2) =   2.9451 [0.2293]
hetero test:       F(6,390) =  0.81962 [0.5551]
hetero-X test:     F(9,387) =  0.89056 [0.5337]
```

The batchfile E10_BFM201.OUT (for PcGive 10.40)

```
---- GiveWin 2.10 session started at 16:26:09 on Monday 06 February 2006 ----

Batch file run: C:\_ects\kurs\fl10\e11_bfm2.fl

Ox version 3.10 (Windows) (C) J.A. Doornik, 1994-2001

Ox version 3.10 (Windows) (C) J.A. Doornik, 1994-2001
---- PcGive 10.40 session started at 16:26:09 on  6-02-2006 ----
bfm201.wk1 loaded from c:\_ects\kurs\wk1\bfm201.wk1

***** Overspecified supply curve
Supply curve:  P = f(C,Q,S), OLS

EQ( 1) Modelling P by OLS (using bfm201.wk1)
       The estimation sample is: 1 to 400

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant           -0.0447211    0.06006   -0.745   0.457   0.0014
Q                   -0.321158    0.07185    -4.47   0.000   0.0479
S                    0.633910    0.07055     8.98   0.000   0.1690
```

```
sigma                 1.19846  RSS               570.211903
R^2                  0.199196  F(2,397) =    49.38 [0.000]**
log-likelihood       -638.484  DW                      1.88
no. of observations       400  no. of parameters          3
mean(P)            -0.0835787  var(P)               1.78012

AR 1-2 test:      F(2,395) =  0.71096 [0.4918]
ARCH 1-1 test:    F(1,395) =   1.7007 [0.1930]
Normality test:   Chi^2(2) =   2.1828 [0.3357]
hetero test:      F(4,392) =   1.0662 [0.3729]
hetero-X test:    F(5,391) =   1.0992 [0.3603]
RESET test:       F(1,396) =   2.2159 [0.1374]
***** Correctly supply curve
Supply curve:  P = f(C,Q), OLS

EQ( 2) Modelling P by OLS (using bfm201.wk1)
       The estimation sample is: 1 to 400

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant           -0.0717624    0.06572    -1.09   0.276   0.0030
Q                    0.187726    0.04844     3.88   0.000   0.0364

sigma                 1.31302  RSS               686.157297
R^2                  0.036363  F(1,398) =    15.02 [0.000]**
log-likelihood       -675.504  DW                      1.92
no. of observations       400  no. of parameters          2
```

```
mean(P)            -0.0835787  var(P)                1.78012

AR 1-2 test:      F(2,396) =   1.0863 [0.3385]
ARCH 1-1 test:    F(1,396) =  0.56753 [0.4517]
Normality test:   Chi^2(2) =   2.1799 [0.3362]
hetero test:      F(2,395) =  0.77404 [0.4618]
hetero-X test:    F(2,395) =  0.77404 [0.4618]
RESET test:       F(1,397) =   5.4813 [0.0197]*
***** Correctly specified price equation
Price equation (RF):  P = f(C,S,D), OLS

EQ( 3) Modelling P by OLS (using bfm201.wk1)
       The estimation sample is: 1 to 400

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant           -0.0570469    0.05996   -0.951   0.342   0.0023
S                    0.386668    0.04332     8.93   0.000   0.1671
D                    0.195448    0.04182     4.67   0.000   0.0521

sigma                 1.19579  RSS               567.679784
R^2                  0.202752  F(2,397) =    50.48 [0.000]**
log-likelihood       -637.594  DW                       1.9
no. of observations       400  no. of parameters          3
mean(P)            -0.0835787  var(P)                1.78012

AR 1-2 test:      F(2,395) =  0.59239 [0.5535]
```

```
ARCH 1-1 test:    F(1,395) = 0.048388 [0.8260]
Normality test:   Chi^2(2) =   3.5801 [0.1670]
hetero test:      F(4,392) =   1.8528 [0.1180]
hetero-X test:    F(5,391) =   1.4930 [0.1910]
RESET test:       F(1,396) =  0.54286 [0.4617]
***** Misspecified price equation
Price equation (RF):  P = f(C,S,D), OLS

EQ( 4) Modelling P by OLS (using bfm201.wk1)
       The estimation sample is: 1 to 400

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant           -0.0476706    0.06147   -0.775   0.439   0.0015
S                    0.385330    0.04444     8.67   0.000   0.1589

sigma                  1.2267  RSS                 598.905929
R^2                  0.158899  F(1,398) =    75.19 [0.000]**
log-likelihood       -648.303  DW                        1.89
no. of observations       400  no. of parameters            2
mean(P)            -0.0835787  var(P)                 1.78012

AR 1-2 test:      F(2,396) =  0.82012 [0.4411]
ARCH 1-1 test:    F(1,396) =  0.84921 [0.3573]
Normality test:   Chi^2(2) =   2.2441 [0.3256]
hetero test:      F(2,395) =   2.7783 [0.0634]
hetero-X test:    F(2,395) =   2.7783 [0.0634]
```

```
RESET test:       F(1,397) =  0.36376 [0.5468]
***** Correctly specified supply curve 2SLS (S and D as instr.)
Supply curve:  P = f(C,Q,S), IVE

EQ( 5) Modelling P by IVE (using bfm201.wk1)
       The estimation sample is: 1 to 400

                  Coefficient  Std.Error  t-value  t-prob
Q              Y     0.500263    0.05746     8.71   0.000
Constant           -0.0520900    0.06909   -0.754   0.451

sigma                 1.37998  RSS               757.923814
Reduced form sigma     1.1958
no. of observations       400  no. of parameters          2
no. endogenous variables    2  no. of instruments         3
mean(P)            -0.0835787  var(P)               1.78012
Additional instruments:
[0] = S
[1] = D

Specification test: Chi^2(1) =0.0068079 [0.9342]
Testing beta = 0:   Chi^2(1) =   75.805 [0.0000]**
***** Correctly specified supply curve 2SLS (S as instr.)
Supply curve:  P = f(C,Q,S), IVE

EQ( 6) Modelling P by IVE (using bfm201.wk1)
```

```
       The estimation sample is: 1 to 400

                  Coefficient  Std.Error  t-value  t-prob
Q              Y     0.497835    0.06454     7.71   0.000
Constant           -0.0522428    0.06907   -0.756   0.450

sigma                 1.37896  RSS                756.813271
Reduced form sigma     1.2267
no. of observations       400  no. of parameters           2
no. endogenous variables    2  no. of instruments          2
mean(P)            -0.0835787  var(P)                1.78012
Additional instruments:
[0] = S
Testing beta = 0:   Chi^2(1) =   59.501 [0.0000]**
***** Correctly specified supply curve 2SLS (D as instr.)
Supply curve:  P = f(C,Q,S), IVE

EQ( 7) Modelling P by IVE (using bfm201.wk1)
       The estimation sample is: 1 to 400

                  Coefficient  Std.Error  t-value  t-prob
Q              Y     0.509627     0.1278     3.99   0.000
Constant           -0.0515006    0.06966   -0.739   0.460

sigma                 1.38394  RSS                762.288664
Reduced form sigma     1.3087
```

```
no. of observations        400  no. of parameters          2
no. endogenous variables     2  no. of instruments         2
mean(P)             -0.0835787  var(P)               1.78012
Additional instruments:
[0] = D
Testing beta = 0:   Chi^2(1) =   15.895 [0.0001]**
***** Overspecified supply curve IV (D as instr.)
Supply curve:  P = f(C,Q), IVE

EQ( 8) Modelling P by IVE (using bfm201.wk1)
       The estimation sample is: 1 to 400

                  Coefficient  Std.Error  t-value  t-prob
Q              Y     0.509475     0.1263     4.03   0.000
S                 -0.00900928     0.1099  -0.0820   0.935
Constant           -0.0523497    0.06945   -0.754   0.451

sigma                 1.38556  RSS               762.155009
Reduced form sigma     1.1958
no. of observations        400  no. of parameters          3
no. endogenous variables     2  no. of instruments         3
mean(P)             -0.0835787  var(P)               1.78012
Additional instruments:
[0] = D
Testing beta = 0:   Chi^2(2) =   75.201 [0.0000]**
***** Overspecified supply curve IV (S as instr.)
```

```
Supply curve:  P = f(C,Q), IVE

EQ( 9) Modelling P by IVE (using bfm201.wk1)
       The estimation sample is: 1 to 400

                  Coefficient  Std.Error  t-value  t-prob
Q              Y     0.497874    0.06440     7.73   0.000
D                  0.00445021    0.05410   0.0823   0.934
Constant           -0.0524566    0.06927   -0.757   0.449

sigma                  1.3807  RSS                 756.81502
Reduced form sigma     1.1958
no. of observations       400  no. of parameters           3
no. endogenous variables    2  no. of instruments          3
mean(P)            -0.0835787  var(P)                1.78012
Additional instruments:
[0] = S
Testing beta = 0:   Chi^2(2) =   75.732 [0.0000]**
***** Correctly specified system
System:  (P,Q) = f(C,S,D), OLS

SYS(10) Estimating the system by OLS (using bfm201.wk1)
        The estimation sample is: 1 to 400

URF equation for: P
                  Coefficient  Std.Error  t-value  t-prob
```

```
S                     0.386668    0.04332     8.93   0.000
D                     0.195448    0.04182     4.67   0.000
Constant       U   -0.0570469    0.05996   -0.951   0.342

sigma = 1.19579   RSS = 567.679784

URF equation for: Q
                  Coefficient  Std.Error  t-value  t-prob
S                     0.776639    0.02285     34.0   0.000
D                     0.383627    0.02206     17.4   0.000
Constant       U  -0.00921979    0.03162   -0.292   0.771

sigma = 0.630654   RSS = 157.8964251

log-likelihood    -960.885156  -T/2log|Omega|     174.265671
|Omega|           0.418395402  log|Y'Y/T|          1.1476647
R^2(LR)              0.867211  R^2(LM)              0.433613
no. of observations       400  no. of parameters           6

F-test on regressors except unrestricted: F(4,792) = 345.355 [0.0000] **
F-tests on retained regressors, F(2,396) =
           S       1028.80 [0.000]**           D       271.191 [0.000]**
    Constant U    0.847674 [0.429]

correlation of URF residuals (standard deviations on diagonal)
                  P            Q
```

```
P             1.1958     -0.50314
Q          -0.50314      0.63065
correlation between actual and fitted
             P             Q
      0.45028      0.88605

P            : Portmanteau(12): 13.2461
Q            : Portmanteau(12): 3.48406
P            : AR 1-2 test:      F(2,395) =  0.59239 [0.5535]
Q            : AR 1-2 test:      F(2,395) =  0.12592 [0.8817]
P            : Normality test:   Chi^2(2) =   3.5801 [0.1670]
Q            : Normality test:   Chi^2(2) =   3.6616 [0.1603]
P            : ARCH 1-1 test:    F(1,395) = 0.048388 [0.8260]
Q            : ARCH 1-1 test:    F(1,395) =   2.1287 [0.1454]
P            : hetero test:      F(4,392) =   1.8528 [0.1180]
Q            : hetero test:      F(4,392) =   1.5919 [0.1756]
P            : hetero-X test:    F(5,391) =   1.4930 [0.1910]
Q            : hetero-X test:    F(5,391) =   1.2832 [0.2703]

Vector Portmanteau(12): 41.3467
Vector AR 1-2 test:      F(8,784) =  0.69054 [0.7002]
Vector Normality test:   Chi^2(4) =   9.6842 [0.0461]*
Vector hetero test:      F(12,1032)=   1.2866 [0.2203]
Vector hetero-X test:    F(15,1074)=   1.0850 [0.3656]
Order condition for P not satisfied!
Order condition for Q not satisfied!
```

```
MOD(11) Estimating the model by FIML (using bfm201.wk1)
        The estimation sample is: 1 to 400

Equation for: P
                  Coefficient  Std.Error  t-value  t-prob
Q                    0.509475     0.1263     4.03   0.000
S                 -0.00900928     0.1099  -0.0820   0.935
Constant       U   -0.0523497    0.06945   -0.754   0.451

sigma = 1.38556

Equation for: Q
                  Coefficient  Std.Error  t-value  t-prob
P                     2.00854     0.2598     7.73   0.000
D                 -0.00893842     0.1092  -0.0819   0.935
Constant       U     0.105361     0.1408    0.748   0.455

sigma = 2.77319

log-likelihood    -960.885156  -T/2log|Omega|     174.265671
no. of observations       400  no. of parameters           6
No restrictions imposed
BFGS using analytical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
```

```
correlation of structural residuals (standard deviations on diagonal)
                  P            Q
P            1.3856     -0.99999
Q          -0.99999       2.7732

P            : Portmanteau(12): 13.2461
Q            : Portmanteau(12): 3.48406
P            : AR 1-2 test:      F(2,395) =  0.59239 [0.5535]
Q            : AR 1-2 test:      F(2,395) =  0.12592 [0.8817]
P            : Normality test:   Chi^2(2) =   3.5801 [0.1670]
Q            : Normality test:   Chi^2(2) =   3.6616 [0.1603]
P            : ARCH 1-1 test:    F(1,395) = 0.048388 [0.8260]
Q            : ARCH 1-1 test:    F(1,395) =   2.1287 [0.1454]
P            : hetero test:      F(4,392) =   1.8528 [0.1180]
Q            : hetero test:      F(4,392) =   1.5919 [0.1756]
P            : hetero-X test:    F(5,391) =   1.4930 [0.1910]
Q            : hetero-X test:    F(5,391) =   1.2832 [0.2703]

Vector Portmanteau(12): 41.3467
Vector EGE-AR 1-2 test:  F(8,784) =  0.69054 [0.7002]
Vector Normality test:   Chi^2(4) =   9.6842 [0.0461]*
Vector hetero test:      F(12,1032)=   1.2866 [0.2203]
Vector hetero-X test:    F(15,1074)=   1.0850 [0.3656]
***** Misspecified model - Instr: S,D

EQ(12) Modelling P by IVE (using bfm201.wk1)
```

```
       The estimation sample is: 1 to 400

                  Coefficient  Std.Error  t-value  t-prob
Q              Y      6.18745      25.13    0.246   0.806
S                    -4.41874      19.53   -0.226   0.821
D                    -2.17822      9.637   -0.226   0.821

sigma                 4.62083  RSS                8476.77149
Reduced form sigma     1.1958
no. of observations       400  no. of parameters           3
no. endogenous variables    2  no. of instruments          3
mean(P)            -0.0835787  var(P)                1.78012
Additional instruments:
[0] = Constant
Testing beta = 0:   Chi^2(2) =   6.7614 [0.0340]*

AR 1-2 test:      F(2,395) = 0.085979 [0.9176]
ARCH 1-1 test:    F(1,395) =   2.5219 [0.1131]
Normality test:   Chi^2(2) =   1.5010 [0.4721]
hetero test:      F(6,390) =   11.187 [0.0000]**
hetero-X test:    F(9,387) =   538.96 [0.0000]**
***** Correctly specified system
System:  (P,Q) = f(C,S,D), OLS

SYS(13) Estimating the system by OLS (using bfm201.wk1)
        The estimation sample is: 1 to 400
```

```
URF equation for: P
                  Coefficient  Std.Error  t-value  t-prob
S                    0.386668    0.04332     8.93   0.000
D                    0.195448    0.04182     4.67   0.000
Constant       U   -0.0570469    0.05996   -0.951   0.342

sigma = 1.19579   RSS = 567.679784

URF equation for: Q
                  Coefficient  Std.Error  t-value  t-prob
S                    0.776639    0.02285     34.0   0.000
D                    0.383627    0.02206     17.4   0.000
Constant       U  -0.00921979    0.03162   -0.292   0.771

sigma = 0.630654   RSS = 157.8964251

log-likelihood    -960.885156  -T/2log|Omega|     174.265671
|Omega|           0.418395402  log|Y'Y/T|          1.1476647
R^2(LR)              0.867211  R^2(LM)              0.433613
no. of observations       400  no. of parameters           6

F-test on regressors except unrestricted: F(4,792) = 345.355 [0.0000] **
F-tests on retained regressors, F(2,396) =
           S       1028.80 [0.000]**           D       271.191 [0.000]**
    Constant U    0.847674 [0.429]
```

```
correlation of URF residuals (standard deviations on diagonal)
                  P             Q
P            1.1958      -0.50314
Q          -0.50314       0.63065
correlation between actual and fitted
            P             Q
      0.45028       0.88605

P           : Portmanteau(12): 13.2461
Q           : Portmanteau(12): 3.48406
P           : AR 1-2 test:      F(2,395) =  0.59239 [0.5535]
Q           : AR 1-2 test:      F(2,395) =  0.12592 [0.8817]
P           : Normality test:   Chi^2(2) =   3.5801 [0.1670]
Q           : Normality test:   Chi^2(2) =   3.6616 [0.1603]
P           : ARCH 1-1 test:    F(1,395) = 0.048388 [0.8260]
Q           : ARCH 1-1 test:    F(1,395) =   2.1287 [0.1454]
P           : hetero test:      F(4,392) =   1.8528 [0.1180]
Q           : hetero test:      F(4,392) =   1.5919 [0.1756]
P           : hetero-X test:    F(5,391) =   1.4930 [0.1910]
Q           : hetero-X test:    F(5,391) =   1.2832 [0.2703]

Vector Portmanteau(12): 41.3467
Vector AR 1-2 test:      F(8,784) =  0.69054 [0.7002]
Vector Normality test:   Chi^2(4) =   9.6842 [0.0461]*
Vector hetero test:      F(12,1032)=   1.2866 [0.2203]
```

```
Vector hetero-X test:    F(15,1074)=   1.0850 [0.3656]
***** Misspecified model - Instr: S

EQ(14) Modelling P by IVE (using bfm201.wk1)
       The estimation sample is: 1 to 400

                  Coefficient  Std.Error  t-value  t-prob
Q              Y      6.18745      25.13    0.246   0.806
S                    -4.41874      19.53   -0.226   0.821
D                    -2.17822      9.637   -0.226   0.821

sigma                 4.62083  RSS                 8476.77149
Reduced form sigma     1.1958
no. of observations       400  no. of parameters            3
no. endogenous variables    2  no. of instruments           3
mean(P)            -0.0835787  var(P)                 1.78012
Additional instruments:
[0] = Constant
Testing beta = 0:   Chi^2(2) =   6.7614 [0.0340]*

AR 1-2 test:      F(2,395) = 0.085979 [0.9176]
ARCH 1-1 test:    F(1,395) =   2.5219 [0.1131]
Normality test:   Chi^2(2) =   1.5010 [0.4721]
hetero test:      F(6,390) =   11.187 [0.0000]**
hetero-X test:    F(9,387) =   538.96 [0.0000]**
```